Calculus III: Practice Midterm I

Name: _________________________________________

• Write your solutions in the space provided. Continue on the back if you need more space.

• You must show your work. Only writing the final answer will receive little credit.

• Partial credit will be given for incomplete work.

• The exam contains 6 problems.

• Good luck!

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>12</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>
1. Write true or false. No justification is needed.
   (a) (2 points) There is a vector \( \mathbf{v} \) such that
   \[
   \mathbf{v} \times \langle 1, 1, 1 \rangle = \langle 1, 2, 3 \rangle.
   \]
   True False

   (b) (2 points) The sixth power of \( 2e^{i\pi/6} \) is a real number.
   True False

   (c) (2 points) The surface described by \( x^2 + y^2 - z^2 = 1 \) is a hyperbolic paraboloid.
   True False

   (d) (2 points) The plane \( 2x + 4y + 6z = 9 \) is perpendicular to the vector \( \langle 1, 1, -1 \rangle \).
   True False
2. Determine whether the following vectors are parallel, perpendicular or neither. Explain why.

(a) (3 points) \( \langle 2, -3, 1 \rangle \) and \( \langle 2, 1, -1 \rangle \).

(b) (3 points) \( 2 \hat{i} + \hat{j} - 4 \hat{k} \) and \( -14 \hat{i} + 7 \hat{j} + 14 \hat{k} \).

(c) (4 points) \( \langle 1, 1, 1 \rangle \) and \( \langle 2, 1, 2 \rangle \times \langle 1, 0, 1 \rangle \).
3. (10 points) Do the four points \((1, 1, 0), (1, 1, -2), (0, 2, -1)\) and \((5, -3, 0)\) lie on the same plane? Justify your answer.
4. (10 points) Find all the complex valued solutions of the equation

\[ x^3 = i. \]

Express your answers both in polar and Cartesian forms.
5. Let $P$ be the plane perpendicular to $\langle 1, 2, 3 \rangle$ and passing through the point $(1, 0, 1)$.

(a) (5 points) Find an equation for $P$.

(b) (5 points) Does the line given by $x(t) = 3t + 1$, $y(t) = 3$ and $z(t) = -t + 3$ intersect the plane $P$?
6. For which (real) values of $a$ are the vectors $\langle 1, a, 2 \rangle$ and $\langle a, 4, 4 \rangle$

(a) (3 points) parallel?

(b) (3 points) perpendicular?

(c) (6 points) For which $a$, does the first vector $\langle 1, a, 2 \rangle$ make an angle of $\pi/4$ with the vector $\mathbf{j}$?