This examination booklet contains 6 problems plus an additional extra credit problem. There are 9 sheets of paper including the front cover and formula sheet. This is a closed book exam. Do all of your work on the pages of this exam booklet. Show all your computations and justify/explain your answers except when indicated. Cross out anything you do not want graded. Calculators are NOT allowed.

You have 75 minutes to complete the exam. Do not begin until instructed to do so. When time is up, stop working and close your test booklet. Cell phones, headphones, laptops and other electronic devices are not allowed.

DO NOT DISCUSS the contents of this exam with anyone until tomorrow, February 15th.

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1. In the following, $\vec{v} = \langle 0, -3, 4 \rangle$ and $\vec{w} = \langle -1, 2, -2 \rangle$.

(a) (4 pts.) What is the vector projection $\text{proj}_{\vec{w}}\vec{v}$?

(b) (4 pts.) What is the cross product $\vec{v} \times \vec{w}$?

(c) (2 pts.) What is the sine of the angle between $\vec{v}$ and $\vec{w}$? (Read carefully!)
2. Multiple choice. Circle the correct answer to each question below. No explanation is needed.

(i) (1 pt.) What is the surface described by the equation

\[ r = 1 \]

written in cylindrical coordinates?

*Hint:* Conversion formulas are available on the last page.

Half of a plane  Sphere  Cylinder  Upwards pointing cone  Downwards pointing cone

(ii) (1 pt.) Suppose that 4 points \( P, Q, R, S \) are given. It is always possible to find a plane containing all four points.

True  False

(iii) (1 pt.) Suppose that \( \vec{u}, \vec{v} \) are two arbitrary vectors in 3-dimensional space. Then we always have the inequality \( |\vec{u} + \vec{v}| \leq |\vec{u}| + |\vec{v}| \).

*Hint:* Draw a picture of the addition law.

True  False

(iv) (1 pt.) Suppose that two lines, \( L_1 \) and \( L_2 \), are given. If \( L_1 \) and \( L_2 \) are not parallel, then there cannot be a plane that contains both \( L_1 \) and \( L_2 \).

True  False

(v) (1 pt.) Suppose that two hyperbolas \( H_1 \) and \( H_2 \) are given, and that \( H_1 \) and \( H_2 \) both have the same pair \( y = \pm x \) of asymptotes. If \( H_1 \) and \( H_2 \) have a point of intersection, then \( H_1 \) and \( H_2 \) are exactly the same hyperbola.

True  False
3.
(a) Multiple choice. Classify the solution sets described by the given equations by circling the name of the surface, or “Nothing” if the equation has no solutions. No explanation is needed.

(i) (1 pt.) $y = -x^2 - z^2$
Ellipsoid  1-Sheeted Hyperboloid  2-Sheeted Hyperboloid  Elliptical Cone
Elliptic Paraboloid  Hyperbolic Paraboloid  Cylinder  Nothing

(ii) (1 pt.) $x^2 - 2y^2 + 4z^2 = -5$
Ellipsoid  1-Sheeted Hyperboloid  2-Sheeted Hyperboloid  Elliptical Cone
Elliptic Paraboloid  Hyperbolic Paraboloid  Cylinder  Nothing

(iii) (1 pt.) $2xy + z^2 = (x + y)^2$
Ellipsoid  1-Sheeted Hyperboloid  2-Sheeted Hyperboloid  Elliptical Cone
Elliptic Paraboloid  Hyperbolic Paraboloid  Cylinder  Nothing

(b) (2 pts.) Suppose that you are only told the following information about the surface $S$: the intersection of $S$ with every plane of the form $z = k$ is an ellipse (and not just a single point or empty), where $k$ is any real number. Which of the following types of surfaces can $S$ be? Circle every possibility. (No explanation needed.)
Ellipsoid  1-Sheeted Hyperboloid  2-Sheeted Hyperboloid  Elliptical Cone
Elliptic Paraboloid  Hyperbolic Paraboloid  Cylinder  Nothing
4.

(a) (4 pts.) What are the focus and directrix of the parabola \( y = x^2 \)?

(b) (6 pts.) If the vertices of an ellipse are at \((5, 0)\) and \((-5, 0)\), and the point \((0, 3)\) is on the ellipse as well, find the equation and foci of the ellipse.
5.

(a) (4 pts.) Find an equation for the sphere $S$ if one of the diameters of $S$ has endpoints $(-5, 3, -1)$ and $(-1, 1, 3)$.

(b) (6 pts.) Find parametric equations for the line of intersection of the planes defined by the equations

\[ 3x + 2y + z = 1 \text{ and } x - y = 1. \]
6.
(a) (5 pts.) Convert the point \((x, y, z) = (0, -1, 0)\) written in rectangular coordinates to spherical coordinates \((\rho, \theta, \phi)\).

*Hint:* Some conversion formulas are available on the last page (if you need them).

(b) (5 pts.) Suppose that you have the following information about the vectors \(\vec{u}, \vec{v}, \vec{w}\).
The angle between \(\vec{u}\) and \(\vec{v}\) is \(\frac{\pi}{2}\) and we have

\[
\vec{u} + \vec{v} + \vec{w} = \vec{0}
\]

\[
|\vec{u}| = \sqrt{3}
\]

\[
|\vec{v}| = 1
\]

What is \(|\vec{w}|\)?
Extra credit. Consider the plane $E$ defined by the equation $x = -1$. Let $P = (1, 0, 0)$.

(a) (4 pts.) Write an equation describing the surface $S$ consisting of all the points $Q$ such that the distance $\text{Dist}(Q, P)$ of $Q$ to $P$ is the same as the distance $\text{Dist}(Q, E)$ of $Q$ to the plane $E$.

*Hint:* The formula sheet below has a formula for the distance between a point and a plane.

(b) (1 pt.) Classify the surface $S$ by circling its name below.

Ellipsoid 1-Sheeted Hyperboloid 2-Sheeted Hyperboloid Elliptic Cone
Elliptic Paraboloid Hyperbolic Paraboloid Cylinder
Feel free to tear off this sheet from the exam.

Conversion from spherical to rectangular coordinates:

\[
x = \rho \sin \phi \cos \theta \\
y = \rho \sin \phi \sin \theta \\
z = \rho \cos \phi
\]

Moreover, in spherical coordinates, the ranges for \( \rho, \phi, \theta \) are:

\[
0 \leq \rho \\
0 \leq \phi \leq \pi \\
0 \leq \theta < 2\pi.
\]

Conversion from cylindrical to rectangular coordinates:

\[
x = r \cos \theta \\
y = r \sin \theta \\
z = z
\]

Formula for the distance from the point \( P \) to the plane \( E \) that has normal vector \( \vec{n} \) and contains the point \( Q \):

\[
\text{Dist}(P, E) = \frac{|\vec{n} \cdot \vec{QP}|}{|\vec{n}|}
\]