Instructions: This exam consists of five true/false and four free response questions; you must solve all problems correctly to receive full credit. Partial credit will be granted only for substantial progress toward a correct solution. No notes, books, or electronic aids are permitted during the exams, and students are not permitted to communicate in any way.

Problem 1. (20 points)
Determine whether each of the following statements is true or false. You may receive limited partial credit for attempting to justify your answer.

(a) True/False: The vector field \( \mathbf{F}(x, y) = (4x, -4y) \) is the gradient of a smooth function on \( \mathbb{R}^2 \).

(b) True/False: There exists a smooth function \( f \) on \( \mathbb{R}^2 \) such that the line integral of \( \nabla f \) along the circle \( x^2 + y^2 = 1 \) is \( 2\pi \).

(c) True/False: For any closed curve \( C \) in \( \mathbb{R}^3 \) and any domain \( D \) which contains \( C \), the line integral of \( \mathbf{F} \) along \( C \) is 0 if \( \mathbf{F} \) is a smooth vector field on \( D \) such that \( \text{curl}(\mathbf{F}) = 0 \).

(d) True/False: If \( \mathbf{F} \) is a smooth vector field on \( \mathbb{R}^3 \) which has a vector potential, then
\[
\int_{S_1} \mathbf{F} \cdot d\mathbf{S} = \int_{S_2} \mathbf{F} \cdot d\mathbf{S}
\]
whenever \( S_1 \) and \( S_2 \) are smooth oriented surfaces with the same smooth oriented boundary curve.

(e) True/False: The vector field \( \mathbf{F}(x, y, z) = (3x \sin y, 3 \cos y + 2xyz, xz^2) \) is the curl of some other vector field.
Problem 2. (20 points)
Let \( \mathbf{F}(x, y) = (6x + e^y, xe^y) \) and let \( C \) be part of the parabola \( y = 1 - x^2 \) in the top half of the plane, oriented clockwise.

(a) Set up, but do not evaluate, a definite integral which calculates \( \int_C \mathbf{F} \cdot d\mathbf{r} \).
(b) Find a potential function for \( \mathbf{F} \).
(c) Use the fundamental theorem of calculus for line integrals to calculate \( \int_C \mathbf{F} \cdot d\mathbf{r} \).

Problem 3. (20 points) Let \( S \) be the surface with parametric representation \( \mathbf{r}(u, v) = (u \cos v, u \sin v, v) \).

0 \leq u \leq 1, 0 \leq v \leq \pi.

(a) Find the equation of the tangent plane to \( S \) at the point \( \mathbf{r}(\frac{1}{2}, \frac{\pi}{2}) \).
(b) Find the area of \( S \).

Problem 4. (20 points)
Let \( C \) be the positively oriented closed curve in the plane consisting of the line segment joining \((-1, 0)\) and \((1, 0)\) and the top half of the circle \( x^2 + y^2 = 1 \).

(a) Set up, but do not evaluate, definite integrals which calculate \( \oint_C xy \, dx + x^2y^2 \, dy \).
(b) Use Green's theorem to calculate \( \oint_C xy \, dx + x^2y^2 \, dy \).

Problem 5. (20 points)
Let \( \mathbf{F}(x, y, z) = (3z, 5x, -2y) \) and let \( C \) be the curve of intersection for the plane \( z = y + 3 \) and the cylinder \( x^2 + y^2 = 1 \), oriented counter-clockwise when viewed from above.

(a) Set up, but do not evaluate, a definite integral which calculates \( \int_C \mathbf{F} \cdot d\mathbf{r} \).
(b) Use Stokes' theorem to calculate \( \int_C \mathbf{F} \cdot d\mathbf{r} \).