

In this work, with Herbert Koch we prove refined estimates for the linearized KdV operator, which allow us to prove scattering for perturbations of solitons in the scaling space appropriate for the quartic nonlinearity, namely $\dot{H}^{-\frac{1}{6}}$. This is an improvement of a work of Tao in that we are able to work purely in the scaling space without additional regularity assumptions.