1. a) Give the definition of compactness.
b) Give the definition of interior point.
c) Define the closure $\bar{E}$ of a set $E \subset X$.
d) Give the definition of countable set.
e) For which $x, y$ in $\mathbb{R}^n$ we have $|x \cdot y| = |x||y|$. (just give the answer without any proof)
f) Give an example of a set which is neither closed nor open.

2. Prove that a perfect set $P$ of $\mathbb{R}^n$ is uncountable.

3. a) Prove that the collection of finite subsets of $\mathbb{N}$ is countable.
b) Prove that the collection of all subsets of $\mathbb{N}$ is uncountable.

4. Show that in $\mathbb{R}^n$ the closure of the open ball $B(x, r)$ is the closed ball
   \[ \bar{B}(x, r) = \{ y \in \mathbb{R}^n, |x - y| \leq r \} \]
   Give an example of metric space for which the corresponding statement is false.

5. Let $F$ be closed and $K$ compact sets of $X$ with $F \cap K = \emptyset$. Show that there exists $\delta > 0$ such that $d(x, y) > \delta$ for all $x \in K$ and $y \in F$.
   Give an example for which the result is not true if $K$ is only closed.