TOPICS IN REPRESENTATION THEORY

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1. Description

This course will be an introduction to the representation theory of infinite-dimensional quantum groups: Yangians, quantum affine algebras and (time permitting) elliptic quantum groups. These objects are related to several diverse branches of mathematics and mathematical physics.

The prerequisites for this course will be kept minimum. Familiarity with the theory of semisimple Lie algebras will be helpful, though not strictly required. No prior knowledge of quantum groups will be assumed.


2. List of topics

2.1. Affine Lie algebras. [16, 22]
(1) Kac–Moody algebras
(2) Affine Kac–Moody algebras: two presentations
(3) Affine root system, affine braid group

2.2. Quantum groups. [14, 21]
(1) Quasi–triangular Hopf algebras
(2) Drinfeld–Jimbo quantized enveloping algebras
(3) Quantum Weyl group
(4) Drinfeld double construction

2.3. Quantum loop algebras. [5, 4, 2, 9, 17, 23]
(1) Two definitions and isomorphism
(2) Classification of irreducible representations
(3) $R$–matrix
(4) $q$–characters
(5) Meromorphic braided tensor categories

2.4. Yangians. [5, 3, 20, 18]
(1) Two definitions and isomorphism
(2) PBW theorem
(3) Classification of irreducible representations
(4) $q$–characters
(5) $R$–matrix
2.5. From quantum loop algebras to Yangians. [13, 10, 11, 12]

(1) Drinfeld’s degeneration homomorphism
(2) Formality
(3) Functor between finite–dimensional representation categories
(4) Drinfeld coproduct and tensor structure

2.6. Elliptic quantum groups. [6, 7, 1, 8, 15, 19]

(1) Dynamical quantum groups and their representations
(2) Felder’s $R$–matrix
(3) Baxter’s $R$–matrix
(4) Two elliptic quantum groups for $\mathfrak{sl}_2$: twist equivalence
(5) Loop presentation
(6) Classification of irreducible representations

REFERENCES

13. N. Guay and X. Ma, From quantum loop algebras to Yangians, doi:10.1112/jlms/jds021 published online.