14. (a) \( x = e^t - 1, \ y = e^{2t}. \)

\[ y = (e^{2t})^2 = (x + 1)^2 \]
and since \( x > -1 \), we have the right side of the
parabola \( y = (x + 1)^2 \).

\[ \text{(b)} \]

18. (a) \( x = \tan^2 \theta, \ y = \sec \theta, -\pi/2 < \theta < \pi/2. \)

\[ 1 + \tan^2 \theta = \sec^2 \theta \quad \Rightarrow \quad 1 + x = y^2 \quad \Rightarrow \quad x = y^2 - 1. \]

For
\(-\pi/2 < \theta < 0, \) we have \( x \geq 0 \) and \( y \geq 1. \) For \( 0 < \theta < \pi/2, \) we have
\( 0 < x \) and \( 1 < y. \) Thus, the curve is the portion of the parabola \( x = y^2 - 1 \)
in the first quadrant. As \( \theta \) increases from \(-\pi/2 \) to \( 0, \) the point \((x, y)\)
approaches \((0, 1)\) along the parabola. As \( \theta \) increases from \( 0 \) to \( \pi/2, \) the
point \((x, y)\) retreats from \((0, 1)\) along the parabola.

\[ \text{(b)} \]

20. \( x = 2 \sin t, \ y = 4 + \cos t \quad \Rightarrow \quad \sin t = \frac{x}{2}, \ \cos t = y - 4. \)

\[ \sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \left(\frac{x}{2}\right)^2 + (y - 4)^2 = 1. \]
The motion
of the particle takes place on an ellipse centered at \((0, 4). \) As \( t \) goes from \( 0 \) to \( \frac{2\pi}{3}, \) the particle starts at the point \((0, 5)\) and
moves clockwise to \((-2, 4) \) [three-quarters of an ellipse].

24. (a) From the first graph, we have \( 1 \leq x \leq 2. \) From the second graph, we have \(-1 \leq y \leq 1. \) The only choice that satisfies
either of those conditions is III.

(b) From the first graph, the values of \( x \) cycle through the values from \(-2 \) to \( 2 \) four times. From the second graph, the values
of \( y \) cycle through the values from \(-2 \) to \( 2 \) six times. Choice I satisfies these conditions.

(c) From the first graph, the values of \( x \) cycle through the values from \(-2 \) to \( 2 \) three times. From the second graph, we have
\( 0 \leq y \leq 2. \) Choice IV satisfies these conditions.

(d) From the first graph, the values of \( x \) cycle through the values from \(-2 \) to \( 2 \) two times. From the second graph, the values of
\( y \) do the same thing. Choice II satisfies these conditions.

42. \( A \) has coordinates \((a \cos \theta, a \sin \theta). \) Since \( OA \) is perpendicular to \( AB, \) \( \triangle OAB \) is a right triangle and \( B \) has coordinates
\((a \sec \theta, 0). \) It follows that \( P \) has coordinates \((a \sec \theta, b \sin \theta). \) Thus, the parametric equations are \( x = a \sec \theta, \ y = b \sin \theta. \)

12. \( x = t^3 + 1, \ y = t^2 - t \quad \Rightarrow \quad \frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{2t - 1}{3t^2} = \frac{2}{3t} - \frac{1}{3t^2} \quad \Rightarrow \]

\[ \frac{d^2y}{dx^2} = \frac{d}{dt} \left( \frac{dy}{dx} \right) = \frac{-2}{3t^2} + \frac{2}{3t^2} = \frac{2(1 - t)}{9t^5}. \]
The curve is CU when \( \frac{d^2y}{dx^2} > 0, \) that is, when \( 0 < t < 1. \)
30. \( x = 3t^2 + 1, y = 2t^3 + 1, \) \( \frac{dx}{dt} = 6t, \frac{dy}{dt} = 6t^2, \) so \( \frac{dy}{dx} = \frac{6t^2}{6t} = t \) [even where \( t = 0 \)].

So at the point corresponding to parameter value \( t \), an equation of the tangent line is \( y - (2t^3 + 1) = t[x - (3t^2 + 1)] \).

If this line is to pass through \((4, 9)\), we must have \(3 - (2t^3 + 1) = t[4 - (3t^2 + 1)] \iff 2t^2 - 2 = 3t - 3t \iff t^3 - 3t + 2 = 0 \iff (t - 1)^2(t + 2) = 0 \iff t = 1 \) or \(-2\). Hence, the desired equations are \( y = x - 4 \), or \( y = x - 1 \), tangent to the curve at \((4, 3)\), and \( y = -15 \) = \(-2(x - 13)\), or \( y = -2x + 11 \), tangent to the curve at \((13, -15)\).

32. The curve \( x = t^2 - 2t = t(t - 2) \), \( y = \sqrt{t} \) intersects the \( y \)-axis when \( x = 0 \), that is, when \( t = 0 \) and \( t = 2 \). The corresponding values of \( y \) are \( 0 \) and \( \sqrt{2} \).

The shaded area is given by

\[
\int_{y=0}^{y=\sqrt{2}} (x_R - x_L) \, dy = \int_{0}^{2} \left[ 0 - x(t) \right] y'(t) \, dt = -\int_{0}^{2} (t^2 - 2t) \left( \frac{1}{2} \sqrt{t} \right) \, dt
\]

\[
= -\int_{0}^{2} \left[ \frac{1}{2} t^{3/2} - t^{1/2} \right] \, dt = -\left[ \frac{1}{5} t^{5/2} - \frac{2}{3} t^{3/2} \right]_{0}^{2}
\]

\[
= -\left( \frac{2}{5} - \frac{4}{3} \right) = -\frac{2}{15}
\]

\[= -\sqrt{2} \cdot \frac{2}{15} = \frac{2}{15} \sqrt{2}
\]

42. \( x = e^t + e^{-t}, \ y = 5 - 2t, \ 0 \leq t \leq 3. \) \( \frac{dx}{dt} = e^t - e^{-t} \) and \( \frac{dy}{dt} = -2, \) so

\( (dx/dt)^2 + (dy/dt)^2 = e^{2t} - 2 + e^{-2t} + 4 = e^{2t} + 2 + e^{-2t} = (e^t + e^{-t})^2. \)

Thus, \( L = \int_{0}^{3} (e^t + e^{-t}) \, dt = [e^t - e^{-t}]_{0}^{3} = e^3 - e^{-3} - (1 - 1) = e^3 - e^{-3}. \)

43. \( x = t \sin t, \ y = t \cos t, \ 0 \leq t \leq 1. \) \( \frac{dx}{dt} = t \cos t + \sin t \) and \( \frac{dy}{dt} = -t \sin t + \cos t, \) so

\( \left( \frac{dx}{dt} \right)^2 + \left( \frac{dy}{dt} \right)^2 = t^2 \cos^2 t + 2t \sin t \cos t + \sin^2 t + t^2 \sin^2 t - 2t \sin t \cos t + \cos^2 t
\]

\[= t^2 (\cos^2 t + \sin^2 t) + \sin^2 t + \cos^2 t = t^2 + 1.
\]

Thus, \( L = \int_{0}^{1} \sqrt{t^2 + 1} \, dt = \left[ \frac{1}{2} t \sqrt{t^2 + 1} + \frac{1}{2} \ln(t + \sqrt{t^2 + 1}) \right]_{0}^{1} = \frac{1}{2} \sqrt{2} + \frac{1}{2} \ln(1 + \sqrt{2}). \)

62. \( x = 3t - t^3, \ y = 3t^2, \ 0 \leq t \leq 1. \) \( (\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 = (3 - 3t^2)^2 + (6t)^2 = 9(1 + t^2 + t^4) = [3(1 + t^2)]^2. \)

\( S = \int_{0}^{1} 2\pi \cdot 3t^2 \cdot 3(1 + t^2) \, dt = 18\pi \int_{0}^{1} (t^2 + t^4) \, dt = 18\pi \left[ \frac{1}{3} t^3 + \frac{1}{5} t^5 \right]_{0}^{1} = \frac{48\pi}{5}. \)

66. \( x = e^t - t, \ y = 4e^{t/2}, \ 0 \leq t \leq 1. \) \( (\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 = (e^t - 1)^2 + (2e^{t/2})^2 = e^{2t} + 2e^t + 1 = (e^t + 1)^2. \)

\( S = \int_{0}^{1} 2\pi (e^t - t) \sqrt{(e^t - 1)^2 + (2e^{t/2})^2} \, dt = \int_{0}^{1} 2\pi (e^t - t)(e^t + 1) \, dt
\]

\[= 2\pi \left[ \frac{1}{2} e^{2t} + e^t - (t - 1)e^t - \frac{1}{2} t^2 \right]_{0}^{1} = \pi(e^2 + 2e - 6). \)
6. (a) $x = 3\sqrt{3}$ and $y = 3$ $\Rightarrow$ $r = \sqrt{(3\sqrt{3})^2 + 3^2} = \sqrt{27 + 9} = 6$ and $\theta = \tan^{-1}\left(\frac{3}{3\sqrt{3}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$. Since $(3\sqrt{3}, 3)$ is in the first quadrant, the polar coordinates are (i) $(6, \frac{\pi}{6})$ and (ii) $(-6, \frac{2\pi}{3})$.

(b) $x = 1$ and $y = -2$ $\Rightarrow$ $r = \sqrt{1^2 + (-2)^2} = \sqrt{5}$ and $\theta = \tan^{-1}\left(-\frac{2}{1}\right) = -\tan^{-1}2$. Since $(1, -2)$ is in the fourth quadrant, the polar coordinates are (i) $(\sqrt{5}, 2\pi - \tan^{-1}2)$ and (ii) $(-\sqrt{5}, \pi - \tan^{-1}2)$.

10. $1 \leq r \leq 3$, $\pi/6 < \theta < 5\pi/6$

![Diagram of a polar region]

$18. \theta = \frac{\pi}{3} \Rightarrow \tan \theta = \tan \frac{\pi}{3} \Rightarrow \frac{y}{x} = \sqrt{3} \Leftrightarrow y = \sqrt{3}x$, a line through the origin.

26. $xy = 4$ $\Leftrightarrow$ $(r \cos \theta)(r \sin \theta) = 4$ $\Leftrightarrow$ $r^2 (\frac{1}{2} \cdot 2 \sin \theta \cos \theta) = 4$ $\Leftrightarrow$ $r^2 \sin 2\theta = 8$ $\Rightarrow$ $r^2 = 8 \csc 2\theta$

58. $r = \cos(\theta/3) \Rightarrow x = r \cos \theta = \cos(\theta/3) \cos \theta, y = r \sin \theta = \cos(\theta/3) \sin \theta \Rightarrow$

\[
\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\cos(\theta/3) \cos \theta + \sin \theta \left(-\frac{1}{3} \sin(\theta/3)\right)}{\cos(\theta/3) (-\sin \theta) + \cos \theta \left(-\frac{1}{3} \sin(\theta/3)\right)}
\]

When $\theta = \pi, \frac{dy}{dx} = \frac{1}{\sqrt{3}/6} = \frac{1}{\sqrt{3}} = -\sqrt{3}$.

64. $r = e^\theta \Rightarrow x = r \cos \theta = e^\theta \cos \theta, y = r \sin \theta = e^\theta \sin \theta \Rightarrow$

\[
\frac{dy}{d\theta} = e^\theta \sin \theta + e^\theta \cos \theta = e^\theta (\sin \theta + \cos \theta) = 0 \Rightarrow \sin \theta = -\cos \theta \Rightarrow \tan \theta = -1 \Rightarrow \theta = -\frac{1}{4} \pi + n\pi \quad [n \text{ any integer}] \Rightarrow \text{horizontal tangents at } \left(e^{\pi(n-1/4)}, \pi(n - \frac{1}{4})\right)
\]

\[
\frac{dx}{d\theta} = e^\theta \cos \theta - e^\theta \sin \theta = e^\theta (\cos \theta - \sin \theta) = 0 \Rightarrow \sin \theta = \cos \theta \Rightarrow \tan \theta = 1 \Rightarrow \theta = \frac{1}{4} \pi + n\pi \quad [n \text{ any integer}] \Rightarrow \text{vertical tangents at } \left(e^{\pi(n+1/4)}, \pi(n + \frac{1}{4})\right).
\]