PRACTICE MID TERM II

(1) (20 points)
(a) \( \frac{d}{dx} \left( \tan^{-1} \left( \sqrt{1 - x^2} \right) \right) \)
(b) \( \frac{d}{dx} \log_5 (2x + 1) \)
(c) \( \lim_{x \to 0} (1 + 3x)^{\cot(x)} \)
(d) \( \frac{d}{dx} (\ln (\sec(x))) \)

(2) (15 points) Find the equation of the line through \((3, 5)\) that cuts off the least area from the first quadrant.

(3) (5 points) Use linear approximation of \(\tan^{-1}(x)\) to find an approximate value of \(\tan^{-1}(1.01)\).

(4) (20 points) True/False. Give proof for the true statements and counterexample for the false ones.
(a) For a function \(f(x)\) and a point \(c\) in the domain of \(f\), \(f'(c) = 0\) implies that \(f(c)\) is either a local maximum or a local minimum.
(b) \(x^3 + x - 1 = 0\) has exactly one solution in the interval \([0, 1]\).
(c) For each \(x\) in the interval \([-1, 1]\) we have \(\sin^{-1}(\cos^{-1}(x)) = \sqrt{1 - x^2}\)
(d) There exists a differentiable function \(f(x)\) (i.e, \(f(x)\) is defined for every \(x\) and \(f'(x)\) exists for every \(x\)) which has no local maximum and exactly two local minima.

(5) (25 points) Consider the function
\[ f(x) = \frac{\sin(x)}{1 + \cos(x)} \]
(a) Find the domain of \(f(x)\).
(b) Find the intervals of increase/decrease, local maxima/minima of \(f(x)\).
(c) Find the intervals of concavity of \(f(x)\) and inflection points.
(d) Find the vertical asymptotes of \(f(x)\).
(e) Use the information of the previous parts to sketch the graph of \(f(x)\) over the domain \(x \in [0, 2\pi]\).
(f) (Bonus 5 points) Sketch the graph of \(f(x)\) over the entire real line.

(6) (10 points) A light source is located at a distance of 50m from a wall. A person walks along a path parallel to the wall, lying in between the light source and the wall, at a distance of 15m from the light source. If the person is walking at a constant speed of 10m/min, at what speed his shadow on the wall is moving?

(7) (5 points) Use Newton’s method to prove that the following iteration computes \(a^{1/3}\):
\[ x_{n+1} = \frac{2}{3} x_n + \frac{a}{3x_n^2} \]