4. \( \mathbf{r}(t) = \langle 2 - t, 4 \sqrt{t} \rangle \) \( \Rightarrow \) At \( t = 1 \):
\[
\mathbf{v}(t) = \mathbf{r}'(t) = \langle -1, 2 / \sqrt{t} \rangle \quad \mathbf{v}(1) = \langle -1, 2 \rangle
\]
\[
\mathbf{a}(t) = \mathbf{r}''(t) = \langle 0, -1/t^{3/2} \rangle \quad \mathbf{a}(1) = \langle 0, -1 \rangle
\]
\[
|\mathbf{v}(t)| = \sqrt{1 + 4/t}
\]

8. \( \mathbf{r}(t) = t \mathbf{i} + 2 \cos t \mathbf{j} + \sin t \mathbf{k} \) \( \Rightarrow \) At \( t = 0 \):
\[
\mathbf{v}(t) = 1 - 2 \sin t \mathbf{j} + \cos t \mathbf{k} \quad \mathbf{v}(0) = \mathbf{i} + \mathbf{k}
\]
\[
\mathbf{a}(t) = -2 \cos t \mathbf{j} - \sin t \mathbf{k} \quad \mathbf{a}(0) = -2 \mathbf{j}
\]
\[
|\mathbf{v}(t)| = \sqrt{1 + 4 \sin^2 t + \cos^2 t} = \sqrt{2 + 3 \sin^2 t}
\]
Since \( y^2/4 + z^2 = 1 \), the path of the particle is an elliptical helix about the \( z \)-axis.

11. \( \mathbf{r}(t) = \sqrt{2} t \mathbf{i} + e^t \mathbf{j} + e^{-t} \mathbf{k} \) \( \Rightarrow \) \( \mathbf{v}(t) = \mathbf{r}'(t) = \sqrt{2} \mathbf{i} + e^t \mathbf{j} - e^{-t} \mathbf{k} \), \( \mathbf{a}(t) = \mathbf{r}'(t) = e^t \mathbf{j} + e^{-t} \mathbf{k} \),
\[
|\mathbf{v}(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}.
\]

16. \( \mathbf{a}(t) = 2 \mathbf{i} + 6t \mathbf{j} + 12t^2 \mathbf{k} \) \( \Rightarrow \) \( \mathbf{v}(t) = \int (2 \mathbf{i} + 6t \mathbf{j} + 12t^2 \mathbf{k}) \, dt = 2t \mathbf{i} + 3t^2 \mathbf{j} + 4t^3 \mathbf{k} + \mathbf{C} \), and \( \mathbf{i} = \mathbf{v}(0) = \mathbf{C} \),
so \( \mathbf{C} = \mathbf{i} \) and \( \mathbf{v}(t) = (2t + 1) \mathbf{i} + 3t^2 \mathbf{j} + 4t^3 \mathbf{k} \). \( \mathbf{r}(t) = \int [(2t + 1) \mathbf{i} + 3t^2 \mathbf{j} + 4t^3 \mathbf{k}] \, dt = (t^2 + t) \mathbf{i} + t^3 \mathbf{j} + t^4 \mathbf{k} + \mathbf{D} \).
But \( t \mathbf{j} + \mathbf{k} = \mathbf{r}(0) = \mathbf{D} \), so \( \mathbf{D} = t \mathbf{j} - \mathbf{k} \) and \( \mathbf{r}(t) = (t^2 + t) \mathbf{i} + (t^3 + 1) \mathbf{j} + (t^4 - 1) \mathbf{k} \).

19. \( \mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle \) \( \Rightarrow \) \( \mathbf{v}(t) = \langle 2t, 5, 2t - 16 \rangle \), \( |\mathbf{v}(t)| = \sqrt{4t^2 + 25 + 4t^2 - 64t + 256} = \sqrt{8t^2 - 64t + 281} \)
and \( \frac{d}{dt} |\mathbf{v}(t)| = \frac{1}{2} (8t^2 - 64t + 281)^{-1/2} (16t - 64) \). This is zero if and only if the numerator is zero, that is,
\[
16t - 64 = 0 \text{ or } t = 4. \text{ Since } \frac{d}{dt} |\mathbf{v}(t)| < 0 \text{ for } t < 4 \text{ and } \frac{d}{dt} |\mathbf{v}(t)| > 0 \text{ for } t > 4, \text{ the minimum speed of } \sqrt{153} \text{ is attained at } t = 4 \text{ units of time.}
\]

22. The argument here is the same as that in Example 13.2.4 with \( \mathbf{r}(t) \) replaced by \( \mathbf{v}(t) \) and \( \mathbf{r}'(t) \) replaced by \( \mathbf{a}(t) \).

27. Let \( \alpha \) be the angle of elevation. Then \( v_0 = 150 \text{ m/s} \) and from Example 5, the horizontal distance traveled by the projectile is
\[
d = \frac{v_0^2 \sin 2\alpha}{g}.
\]
Thus \( \frac{150^2 \sin 2\alpha}{g} = 800 \Rightarrow \sin 2\alpha = \frac{800g}{150^2} \approx 0.3484 \Rightarrow 2\alpha \approx 20.4^\circ \text{ or } 180 - 20.4 = 159.6^\circ.
\]

Two angles of elevation then are \( \alpha \approx 10.2^\circ \) and \( \alpha \approx 79.8^\circ \).
36. From Equation 7 we have \( \alpha = v'T + \kappa v^2 N \). If a particle moves along a straight line, then \( \kappa = 0 \) [see Section 13.3], so the acceleration vector becomes \( \alpha = v'T \). Because the acceleration vector is a scalar multiple of the unit tangent vector, it is parallel to the tangent vector.

(b) If the speed of the particle is constant, then \( v' = 0 \) and Equation 7 gives \( \alpha = \kappa v^2 N \). Thus the acceleration vector is parallel to the unit normal vector (which is perpendicular to the tangent vector and points in the direction that the curve is turning).

39. \( r(t) = \cos t \mathbf{i} + \sin t \mathbf{j} + t \mathbf{k} \Rightarrow r'(t) = -\sin t \mathbf{i} + \cos t \mathbf{j} + \mathbf{k}, \quad |r'(t)| = \sqrt{\sin^2 t + \cos^2 t + 1} = \sqrt{2}, \)
\( r''(t) = -\cos t \mathbf{i} - \sin t \mathbf{j}, \quad r'(t) \times r''(t) = \sin t \mathbf{i} - \cos t \mathbf{j} + \mathbf{k}. \)

Then \( a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} = \frac{\sin t \cos t - \sin t \cos t}{\sqrt{2}} = 0 \) and \( a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{\sqrt{\sin^2 t + \cos^2 t + 1}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1. \)

40. \( r(t) = t \mathbf{i} + t^2 \mathbf{j} + 3t \mathbf{k} \Rightarrow r'(t) = 2t \mathbf{j} + 3 \mathbf{k}, \quad |r'(t)| = \sqrt{1 + (2t)^2 + 3^2} = \sqrt{4t^2 + 10}, \)
\( r''(t) = 2 \mathbf{j}, \quad r'(t) \times r''(t) = -6 \mathbf{i} + 2 \mathbf{k}. \)

Then \( a_T = \frac{r'(t) \cdot r''(t)}{|r'(t)|} = \frac{4t}{\sqrt{4t^2 + 10}} \) and \( a_N = \frac{|r'(t) \times r''(t)|}{|r'(t)|} = \frac{2 \sqrt{10}}{\sqrt{4t^2 + 10}}. \)

17. \( \sqrt{1 - x^2} \) is defined only when \( 1 - x^2 \geq 0 \), or
\[ x^2 \leq 1 \iff -1 \leq x \leq 1, \text{ and } \sqrt{1 - y^2} \text{ is defined only when } 1 - y^2 \geq 0, \text{ or } y^2 \leq 1 \iff -1 \leq y \leq 1. \]

Thus the domain of \( f \) is
\[ \{(x, y) \mid -1 \leq x \leq 1, \ -1 \leq y \leq 1\}. \]
25. \( z = 10 - 4x - 5y \) or \( 4x + 5y + z = 10 \), a plane with intercepts 2, 5, 2, and 10.

28. \( z = 1 + 2x^2 + 2y^2 \), a circular paraboloid with vertex at \((0, 0, 1)\).
32. All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x, y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x, y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

(c) $f(x, y) = \frac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \frac{1}{1 + y^2}$, and in $y = 0$ is $z = \frac{1}{1 + x^2}$. In addition, we can see that $f$ is close to 0 for large values of $x$ and $y$, so this is graph I.

(d) $f(x, y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \Rightarrow y = \pm x$, so it must be graph IV.

(e) $f(x, y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \Rightarrow y = x$, so it must be graph II.

(f) $f(x, y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.

59. $z = \sin(xy)$

(a) C

(b) II

Reasons: This function is periodic in both $x$ and $y$, and the function is the same when $x$ is interchanged with $y$, so its graph is symmetric about the plane $y = x$. In addition, the function is 0 along the $x$- and $y$-axes. These conditions are satisfied only by C and II.

60. $z = e^x \cos y$

(a) A

(b) IV

Reasons: This function is periodic in $y$ but not $x$, a condition satisfied only by A and IV. Also, note that traces in $x = k$ are cosine curves with amplitude that increases as $x$ increases.

61. $z = \sin(x - y)$

(a) F

(b) I

Reasons: This function is periodic in both $x$ and $y$ but is constant along the lines $y = x + k$, a condition satisfied only by F and I.

62. $z = \sin x - \sin y$

(a) E

(b) III

Reasons: This function is periodic in both $x$ and $y$, but unlike the function in Exercise 61, it is not constant along lines such as $y = x + \pi$, so the contour map is III. Also notice that traces in $y = k$ are vertically shifted copies of the sine wave $z = \sin x$, so the graph must be E.
63. \( z = (1 - x^2)(1 - y^2) \)  
   (a) B   (b) VI  
   Reasons: This function is 0 along the lines \( x = \pm 1 \) and \( y = \pm 1 \). The only contour map in which this could occur is VI. Also note that the trace in the \( xz \)-plane is the parabola \( z = 1 - x^2 \) and the trace in the \( yz \)-plane is the parabola \( z = 1 - y^2 \), so the graph is B.

64. \( z = \frac{x - y}{1 + x^2 + y^2} \)  
   (a) D   (b) V  
   Reasons: This function is not periodic, ruling out the graphs in A, C, E, and F. Also, the values of \( z \) approach 0 as we use points farther from the origin. The only graph that shows this behavior is D, which corresponds to V.

7. \( f(x, y) = \frac{4 - xy}{x^2 + 3y^2} \) is a rational function and hence continuous on its domain.

\[ (2, 1) \text{ is in the domain of } f \text{, so } f \text{ is continuous there and } \lim_{(x,y)\to(2,1)} f(x,y) = f(2,1) = \frac{4 - (2)(1)}{(2)^2 + 3(1)^2} = \frac{2}{7}. \]

10. \( f(x, y) = \frac{5y^4 \cos^2 x}{x^4 + y^4} \). First approach \((0, 0)\) along the \( x \)-axis. Then \( f(x, 0) = \frac{0}{x^4} = 0 \) for \( x \neq 0 \), so \( f(x, y) \to 0 \). Next approach \((0, 0)\) along the \( y \)-axis. For \( y \neq 0 \), \( f(0, y) = \frac{5y^4}{y^4} = 5 \), so \( f(x, y) \to 5 \). Since \( f \) has two different limits along two different lines, the limit does not exist.

13. \( f(x, y) = \frac{xy}{\sqrt{x^2 + y^2}} \). We can see that the limit along any line through \((0, 0)\) is 0, as well as along other paths through \((0, 0)\) such as \( x = y^2 \) and \( y = x^2 \). So we suspect that the limit exists and equals 0, we use the Squeeze Theorem to prove our assertion. \( 0 \leq \left| \frac{xy}{\sqrt{x^2 + y^2}} \right| \leq |x| \) since \( |y| \leq \sqrt{x^2 + y^2} \), and \( |x| \to 0 \) as \((x,y)\to(0,0)\). So \( \lim_{(x,y)\to(0,0)} f(x,y) = 0 \).

14. \( f(x, y) = \frac{x^4 - y^4}{x^2 + y^2} = \frac{(x^2 + y^2)(x^2 - y^2)}{x^2 + y^2} = x^2 - y^2 \) for \((x,y) \neq (0,0)\). Thus the limit as \((x,y)\to(0,0)\) is 0.

15. Let \( f(x, y) = \frac{x^2 ye^y}{x^4 + 4y^2} \). Then \( f(x, 0) = 0 \) for \( x \neq 0 \), so \( f(x, y) \to 0 \) as \((x,y)\to(0,0)\) along the \( x \)-axis. Approaching \((0, 0)\) along the \( y \)-axis or the line \( y = x \) also gives a limit of 0. But \( f(x, x^2) = \frac{x^2 x^2 e^{x^2}}{x^4 + 4(x^2)^2} = \frac{x^4 e^{x^2}}{5x^4} = \frac{e^{x^2}}{5} \) for \( x \neq 0 \), so \( f(x, y) \to e^0/5 = \frac{1}{5} \) as \((x,y)\to(0,0)\) along the parabola \( y = x^2 \). Thus the limit doesn’t exist.
18. \( f(x, y) = xy^4/(x^2 + y^8) \). On the \( x \)-axis, \( f(x, 0) = 0 \) for \( x \neq 0 \), so \( f(x, y) \to 0 \) as \( (x, y) \to (0, 0) \) along the \( x \)-axis.

Approaching \( (0, 0) \) along the curve \( x = y^4 \) gives \( f(y^4, y) = y^6/2y^8 = \frac{1}{2} \) for \( y \neq 0 \), so along this path \( f(x, y) \to \frac{1}{2} \) as \( (x, y) \to (0, 0) \). Thus the limit does not exist.

31. \( F(x, y) = \frac{1 + x^2 + y^2}{1 - x^2 - y^2} \) is a rational function and thus is continuous on its domain

\[ \{(x, y) \mid 1 - x^2 - y^2 \neq 0\} = \{(x, y) \mid x^2 + y^2 \neq 1\} . \]

37. \( f(x, y) = \begin{cases} \frac{x^2y^3}{2x^2 + y^2} & \text{if } (x, y) \neq (0, 0) \\ 1 & \text{if } (x, y) = (0, 0) \end{cases} \)

The first piece of \( f \) is a rational function defined everywhere except at the origin, so \( f \) is continuous on \( \mathbb{R}^2 \) except possibly at the origin. Since \( x^2 \leq 2x^2 + y^2 \), we have \( |x^2y^3/(2x^2 + y^2)| \leq |y^3| \). We know that \( |y^3| \to 0 \) as \( (x, y) \to (0, 0) \). So, by the Squeeze Theorem,

\[ \lim_{(x, y) \to (0, 0)} f(x, y) = \lim_{(x, y) \to (0, 0)} \frac{x^2y^3}{2x^2 + y^2} = 0. \]

But \( f(0, 0) = 1 \), so \( f \) is discontinuous at \((0, 0)\). Therefore, \( f \) is continuous on the set \( \{(x, y) \mid (x, y) \neq (0, 0)\} \).