PRACTICE FINAL
CALCULUS III

(1) • Find the volume of the parallelopiped formed by:
\( \vec{u} = \langle 1, 0, 2 \rangle \quad \vec{v} = \langle 2, -1, 0 \rangle \quad \vec{w} = \langle 4, 1, 1 \rangle \)

• Find the parametric equations describing the tangent line to the following parametric curve, at \( (2, -4, 3) \)
\( \vec{r}(t) = \langle \sqrt{2}t, 4 - t^2, t^2 - 1 \rangle \)

• Write \( w = \frac{1 + \sqrt{3}i}{1 - \sqrt{3}i} \) in \( a + bi \) form.

• Find the length of the following curve
\( \vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle \quad 0 \leq t \leq 1 \)

• Compute the following limit, or prove that it doesn’t exist.
\( \lim_{(x,y) \to (0,0)} \frac{5x \sin^2(y)}{x^2 + y^4} \)

(2) Let \( f(x, y) = \ln(1 + x^2 + y^2) \).

• Compute \( f_{xx} \) and \( f_{xy} \).

• Write the unit vector along which \( f \) is increasing fastest at \( x = y = 1 \).

• What is the rate of change of \( f \) at \( (1, 1) \) in the direction of \( \langle 1, -2 \rangle \).

(3) Find all critical points of \( f(x, y) = 2x^3 + y^3 - 5xy \) and classify them as local minimum, local maximum, saddle point.

(4) Find the absolute maximum and minimum values of \( f(x, y) = xy - 5x^2 + 3 \) on the domain \( D \) bounded by \( x \)-axis, \( x = 2 \) and \( y = x^3 \).

(5) Let \( C \) be the curve of intersection of the following two surfaces
\( x^2 + y^2 = 1 \quad \text{and} \quad z = 3 - 2x^2 - 4y^2 \)
Find points on \( C \) which are closest to and furthest from the origin.

(6) A projectile is fired with an initial speed of 100 m/s at an angle of 60°.

• Write the position and velocity of the projectile as a function of \( t \).

• At what times is the projectile at a height three quarters of its maximum height.
(7) Assume that $z$ is implicitly defined as a function of $x, y$ by
\[ \cos(yz) + x^2z = 9 \]
If at $x = 2, y = 0, z = 2$, the value of $x$ starts increasing at a rate of 1 unit per second, and the value of $y$ starts decreasing at a rate of 2 units per second, compute the rate of change of $z$.

(8) Let $\vec{r}(t)$ be a parametric curve. Prove that
\[ \frac{d}{dt} \left( \frac{\vec{r}(t)}{||\vec{r}(t)||} \right) = \frac{1}{||\vec{r}(t)||} \left( \vec{r}'(t) - \text{Proj}_{\vec{r}(t)}(\vec{r}'(t)) \right) \]

(9) Find the distance between the point $(1, 3, 2)$ and the line
\[ \frac{x - 5}{4} = y = \frac{z - 1}{2} \]