• The use of class notes, book, formulae sheet, calculator is not permitted.

• In order to get full credit, you **must** show all your work.

• Please write solution to a problem in the space provided.

• You have **two hours and fifty minutes**.

• Do not forget to write your name and UNI in the space provided below.

Print UNI ______________________
Print Name ______________________
Section ______________________

For Grader’s use only:

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Problem 1 (16 points)

(a) Let \( f(x, y) \) be a function of two variables such that the following directional derivatives are known at \((0, 0)\).

\[
D_{\frac{1}{\sqrt{2}}} f(0, 0) = -\sqrt{2} \quad D_{\frac{1}{\sqrt{5}}} f(0, 0) = \sqrt{5}
\]

Compute the gradient of \( f(x, y) \) at \((0, 0)\).

(b) Prove that

\[
\lim_{(x,y) \to (0,0)} \frac{2xy^5}{x^2 + y^8} = 0
\]
(c) Find all (complex) solutions to the following equation:

\[ z^3 - z^2 + z - 1 = 0 \]

(d) Compute the three cuberoots of \( z = -1 + \sqrt{3} \, i \).
Problem 2 (10 points) The plane $4x - 3y + 8z = 5$ intersects the cone $z^2 = x^2 + y^2$ in an ellipse. Use Lagrange multipliers to find the highest and lowest points on this ellipse. (Highest/lowest = largest/smallest $z$-coordinate).
Problem 3 (12 points) Let $F(x, y, z) = \ln(1 + z(x^2 - y^3))$.
(a) Compute the differential of $F$.

(b) Find the direction in which $F$ increases most rapidly at $(1, 1, 3)$. What is the fastest rate of increase?

(c) Use linear approximation at $(1, 1, 3)$ to find the approximate value of $F(0.95, 0.95, 3.01)$. 

Problem 4 (8 points) A projectile is fired from a point 80 m above the ground, with an initial speed of 60 m/s at an angle of 30°. (use acceleration due to gravity $g = 10 \text{ m/s}^2$).

(a) Write the position $\vec{r}(t)$ and velocity $\vec{v}(t)$ at time $t$.

(b) Where does the projectile hit the ground?
**Problem 5** (10 points) Find the absolute maximum and minimum values of the function
\[ f(x, y) = x^2 + y^2 - 2y \]
on the region bounded by the triangle with vertices \((2, 0), (0, 2)\) and \((-2, 0)\).
Problem 6 (10 points) Find the critical points of \( f(x, y) = y^3 + 3x^2y - 6x^2 - 6y^2 + 2 \) and use the second derivative test to classify them as local maxima, local minima or saddle point.
Problem 7 (16 points)

(a) Find the parametric equations describing the intersection of the following two surfaces

\[ z = \sqrt{x^2 + y^2} \quad \text{and} \quad z = 1 + y \]

(b) Find the length of the following parametric curve:

\[ \mathbf{r}(t) = \left\langle 9t, (2t)^{3/2}, \frac{t^2}{2} \right\rangle \quad 0 \leq t \leq 2 \]
(c) Find the rate of change of the volume of a rectangular box, if its length and width are increasing at a rate of 1 cm/s while its height is decreasing at a rate of 2 cm/s, when its length and width are 20 cm and its height is 10 cm.

(d) Write the equation of the tangent plane to $xy + yz + xz = 3$ at the point $(1, 1, 1)$. 
Problem 8 (8 points) Let $\vec{r}(t) = \langle t, 2t, t^2 \rangle$ be the parametric curve describing the path of a particle.

(a) Compute the tangential and normal components of its acceleration.

(b) Find the curvature at time $t = 2$. 