PRACTICE MID TERM I
CALCULUS III

- The use of class notes, book, formulae sheet, calculator is not permitted.

- In order to get full credit, you must show all your work.

- Each solution must have a clearly labeled problem number and start at the top of a new page.

- You have one hour and fifteen minutes.

- Do not forget to write your name and UNI in the space provided below and on the notebook provided.
(1) Write the parametric equations describing the line
- lying on the plane \( P : x - 2y + z = 4 \)
- passing through the intersection of the plane \( P \) with the line \( L : \vec{r} = (1 + t, 1 - t, 2t) \), and
- perpendicular to line \( L \).

(2) Let \( A = (1, 0, 1) \), \( B = (3, 1, 0) \) and \( C = (3, 2, 2) \) and \( D = (-2, -2, 1) \) be four points in \( \mathbb{R}^3 \).
   (a) Find the volume of the parallelopiped formed by edges \( AB, AC \) and \( AD \).
   (b) Find the coordinates of the point \( E \) opposite to \( A \) in this parallelopiped.
   (c) Find the angle \( \angle EAB \).

(3) Consider the plane \( P : x + y - 3z = 1 \) and a point \( P = (2, 1, 0) \).
   (a) Find the parametric equations describing the line through \( P \) and perpendicular to the plane \( P \).
   (b) Find the coordinates of the point \( Q \) where this line meets the plane \( P \).
   (c) Compute the length of the line segment \( PQ \).
   (d) Compute the distance between the point \( P \) and the plane \( P \) directly (using the formula) and verify your answer from part (c).

(4) True/False. Justify your answer with a proof if true, or a counterexample if false.
   (a) If \( \vec{v} \cdot \vec{w} = 0 \) then either \( \vec{v} = 0 \) or \( \vec{w} = 0 \).
   (b) If \( \vec{v} \times \vec{w} = 0 \) then \( \vec{v} \) and \( \vec{w} \) must be parallel.
   (c) A pair of lines is either parallel or intersect in a point.
   (d) \( \vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c} \).
   (e) \( |\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}| \).

(5) Consider the equation \( z = ax^2 + y^2 \).
   (a) Sketch the traces for \( a = -1, 0, 1 \).
   (b) Sketch the surface for \( a = -1, 0, 1 \).
   (c) Describe how the surface changes when \( a \) approaches 0 from the left and from the right.

(6) Find the equation of the plane which contains the following two parallel lines:

\[
\begin{align*}
x &= 2 - t \\
y &= 3 + 2t \\
z &= 1 + t
\end{align*}
\]

\[
1 - x = \frac{y - 4}{2} = z - 3
\]

(7) Let \( \vec{v} \) be a non–zero vector.
   - Prove that \( \text{Proj}_{\vec{v}}(\vec{u}) = 0 \) if, and only if \( \vec{u} \) is orthogonal to \( \vec{v} \).
   - Prove that \( \text{Proj}_{\vec{v}}(\vec{u}) = \vec{u} \) if, and only if \( \vec{u} \) is parallel to \( \vec{v} \).