4. The projection of $(2, 3, 5)$ onto the $xy$-plane is $(2, 3, 0)$, 
onto the $yz$-plane, $(0, 3, 5)$; onto the $xz$-plane, $(2, 0, 5)$.

The length of the diagonal of the box is the distance between
the origin and $(2, 3, 5)$, given by
$$\sqrt{(2 - 0)^2 + (3 - 0)^2 + (5 - 0)^2} = \sqrt{38} \approx 6.16$$

12. An equation of the sphere with center $(2, -6, 4)$ and radius 5 is $(x - 2)^2 + [y - (-6)]^2 + (z - 4)^2 = 5^2$ or
$(x - 2)^2 + (y + 6)^2 + (z - 4)^2 = 25$. The intersection of this sphere with the $xy$-plane is the set of points on the sphere
whose $z$-coordinate is 0. Putting $z = 0$ into the equation, we have $(x - 2)^2 + (y + 6)^2 = 9, z = 0$ which represents a circle
in the $xy$-plane with center $(2, -6, 0)$ and radius 3. To find the intersection with the $xz$ plane, we set $y = 0$:
$(x - 2)^2 + (z - 4)^2 = -11$. Since no points satisfy this equation, the sphere does not intersect the $xz$-plane. (Also note that the distance from the center of the sphere to the $xz$-plane is greater than the radius of the sphere.) To find the intersection with the $yz$-plane, we set $x = 0$: $(y + 6)^2 + (z - 4)^2 = 21, x = 0$, a circle in the $yz$-plane with center $(0, -6, 4)$ and radius $\sqrt{21}$.

13. The radius of the sphere is the distance between $(4, 3, -1)$ and $(3, 8, 1): r = \sqrt{(3 - 4)^2 + (8 - 3)^2 + [1 - (-1)]^2} = \sqrt{30}$.
Thus, an equation of the sphere is $(x - 3)^2 + (y - 8)^2 + (z - 1)^2 = 30$.

16. Completing squares in the equation gives $(x^2 + 8x + 16) + (y^2 - 6y + 9) + (z^2 + 2z + 1) = -17 + 16 + 9 + 1 \Rightarrow
(x + 4)^2 + (y - 3)^2 + (z + 1)^2 = 9$, which we recognize as an equation of a sphere with center $(-4, 3, -1)$ and radius 3.

36. For any point on or above the disk in the $xy$-plane with center the origin and radius 2 we have $x^2 + y^2 \leq 4$. Also each point
lies on or between the planes $z = 0$ and $z = 8$, so the region is described by $x^2 + y^2 \leq 4, 0 \leq z \leq 8$.

20. $a + b = (4i + j) + (i - 2j) = 5i - j$

$2a + 3b = 2(4i + j) + 3(1 - 2j) = 8i + 2j + 3i - 6j = 11i - 4j$

$|a| = \sqrt{4^2 + 1^2} = \sqrt{17}$

$|a - b| = |(4i + j) - (i - 2j)| = |3i + 3j| = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$
22. \(a + b = (2i - 4j + 4k) + (2j - k) = 2i - 2j + 3k\)

\[2a + 3b = 2(2i - 4j + 4k) + 3(2j - k) = 4i - 8j + 8k + 6j - 3k = 4i - 2j + 5k\]

\[|a| = \sqrt{2^2 + (-4)^2 + 4^2} = \sqrt{36} = 6\]

\[|a - b| = |(2i - 4j + 4k) - (2j - k)| = |2i - 6j + 5k| = \sqrt{2^2 + (-6)^2 + 5^2} = \sqrt{65}\]

30. From the figure, we see that the horizontal component of the force \(F\) is \(|F| \cos 38^\circ = 50 \cos 38^\circ \approx 39.4\) N, and the vertical component is \(|F| \sin 38^\circ = 50 \sin 38^\circ \approx 30.8\) N.

44. \(\overrightarrow{AC} = \frac{1}{2} \overrightarrow{AB}\) and \(\overrightarrow{BC} = \frac{1}{2} \overrightarrow{BA} \Rightarrow \overrightarrow{AB} = 3\overrightarrow{c} - 3\overrightarrow{a}, \overrightarrow{c} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OA} + \frac{1}{3} \overrightarrow{BA} \Rightarrow\)

\(\overrightarrow{BA} = \frac{2}{3} \overrightarrow{c} - \frac{2}{3} \overrightarrow{b}, \overrightarrow{BA} = -\overrightarrow{AB}, \text{ so } \frac{2}{3} \overrightarrow{c} - \frac{2}{3} \overrightarrow{b} = 3\overrightarrow{a} - 3\overrightarrow{c} \Rightarrow \overrightarrow{c} + 2\overrightarrow{c} = 2\overrightarrow{a} + \overrightarrow{b} \Rightarrow \overrightarrow{c} = \frac{2}{3} \overrightarrow{a} + \frac{1}{3} \overrightarrow{b}.

1. (a) \(a \cdot b\) is a scalar, and the dot product is defined only for vectors, so \((a \cdot b) \cdot c\) has no meaning.

(b) \((a \cdot b)\) is a scalar multiple of a vector, so it does have meaning.

(c) Both \(|a|\) and \(b \cdot c\) are scalars, so \(|a| (b \cdot c)\) is an ordinary product of real numbers, and has meaning.

(d) Both \(a\) and \(b + c\) are vectors, so the dot product \(a \cdot (b + c)\) has meaning.

(e) \(a \cdot b\) is a scalar, but \(c\) is a vector, and so the two quantities cannot be added and \(a \cdot b + c\) has no meaning.

(f) \(|a|\) is a scalar, and the dot product is defined only for vectors, so \(|a| \cdot (b + c)\) has no meaning.

5. \(a \cdot b = \langle 4, 1, \frac{1}{4} \rangle \cdot \langle 6, -3, -8 \rangle = (4)(6) + (1)(-3) + \left(\frac{1}{4}\right)(-8) = 19\)

10. By Theorem 3, \(a \cdot b = |a||b| \cos \theta = (3\sqrt{6})(\cos 45^\circ) = 3\sqrt{6}\left(\frac{\sqrt{2}}{2}\right) = \frac{3}{2} \cdot 2\sqrt{3} = 3\sqrt{3} \approx 5.20\).

24. (a) Because \(u = -\frac{3}{4}v\), \(u\) and \(v\) are parallel vectors (and thus not orthogonal).

(b) \(u \cdot v = (1)(2) + (-1)(-1) + (2)(1) = 5 \neq 0\), so \(u\) and \(v\) are not orthogonal. Also, \(u\) is not a scalar multiple of \(v\), so \(u\) and \(v\) are not parallel.

(c) \(u \cdot v = (a)(-b) + (a)(c)(0) = -ab + ab + 0 = 0\), so \(u\) and \(v\) are orthogonal (and not parallel).

42. \(|a| = \sqrt{4 + 9 + 36} = 7\) so the scalar projection of \(b\) onto \(a\) is \(\text{comp}_a b = \frac{a \cdot b}{|a|} = \frac{1}{7}(-10 - 3 - 24) = -\frac{37}{7}\), while the vector projection is \(\text{proj}_a b = \frac{\frac{37}{7} \overrightarrow{a}}{|a|} = -\frac{37}{7} \cdot \frac{1}{7} \langle -2, 3, -6 \rangle = -\frac{37}{49} \langle -2, 3, -6 \rangle = \langle \frac{74}{49}, -\frac{111}{49}, \frac{222}{49} \rangle\).
53. First note that \( \mathbf{n} = (a, b) \) is perpendicular to the line, because if \( Q_1 = (a_1, b_1) \) and \( Q_2 = (a_2, b_2) \) lie on the line, then
\[
\mathbf{n} \cdot \mathbf{Q}_1 \mathbf{Q}_2 = aa_2 - aa_1 + bb_2 - bb_1 = 0,
\]
since \( aa_2 + bb_2 = -c = aa_1 + bb_1 \) from the equation of the line.

Let \( P_2 = (x_2, y_2) \) lie on the line. Then the distance from \( P_1 \) to the line is the absolute value of the scalar projection
\[
\text{comp}_n (\mathbf{P}_1 \mathbf{P}_2) = \frac{|\mathbf{n} \cdot (x_2 - x_1, y_2 - y_1)|}{|\mathbf{n}|} = \frac{|ax_2 - ax_1 + by_2 - by_1|}{\sqrt{a^2 + b^2}} = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}
\]
since \( ax_2 + by_2 = -c \). The required distance is
\[
\frac{|(3)(-2) + (-4)(3) + 5|}{\sqrt{3^2 + (-4)^2}} = \frac{13}{5}.
\]

56. Consider a cube with sides of unit length, wholly within the first octant and with edges along each of the three coordinate axes. \( \mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( \mathbf{i} + \mathbf{j} \) are vector representations of a diagonal of the cube and a diagonal of one of its faces. If \( \theta \) is the angle between these diagonals, then
\[
\cos \theta = \frac{(\mathbf{i} + \mathbf{j} + \mathbf{k}) \cdot (\mathbf{i} + \mathbf{j})}{|\mathbf{i} + \mathbf{j} + \mathbf{k}| |\mathbf{i} + \mathbf{j}|} = \frac{1 + 1}{\sqrt{3} \sqrt{2}} = \sqrt{\frac{2}{3}} \Rightarrow \theta = \cos^{-1} \sqrt{\frac{2}{3}} \approx 35^\circ.
\]

4. \( \mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ 0 & 1 & 7 \\ 2 & -1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 7 \\ -1 & 4 \end{vmatrix} \mathbf{i} - \begin{vmatrix} 0 & 7 \\ 2 & 4 \end{vmatrix} \mathbf{j} + \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix} \mathbf{k} = [4 - (-7)] \mathbf{i} - [0 - 14] \mathbf{j} + [0 - 2] \mathbf{k} = 11 \mathbf{i} + 14 \mathbf{j} - 2 \mathbf{k}
\]

Since \( (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = (11 \mathbf{i} + 14 \mathbf{j} - 2 \mathbf{k}) \cdot (\mathbf{j} + 7 \mathbf{k}) = 0 + 14 - 14 = 0 \), \( \mathbf{a} \times \mathbf{b} \) is orthogonal to \( \mathbf{a} \).

Since \( (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = (11 \mathbf{i} + 14 \mathbf{j} - 2 \mathbf{k}) \cdot (2 \mathbf{i} - \mathbf{j} + 4 \mathbf{k}) = 22 - 14 - 8 = 0 \), \( \mathbf{a} \times \mathbf{b} \) is orthogonal to \( \mathbf{b} \).

5. \( \mathbf{a} \times \mathbf{b} = \begin{vmatrix} i & j & k \\ 1 & -1 & -1 \\ \frac{1}{2} & 1 & \frac{1}{2} \end{vmatrix} = \begin{vmatrix} -1 & -1 \\ 1 & \frac{1}{2} \end{vmatrix} \mathbf{i} - \begin{vmatrix} 1 & -1 \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} \mathbf{j} + \begin{vmatrix} 1 & -1 \\ \frac{1}{2} & 1 \end{vmatrix} \mathbf{k} = [-\frac{1}{2} - (-1)] \mathbf{i} - [\frac{1}{2} - (-\frac{1}{2})] \mathbf{j} + [1 - (-1)] \mathbf{k} = \frac{1}{2} \mathbf{i} - \frac{1}{2} \mathbf{j} + \frac{1}{2} \mathbf{k}
\]

Now \( (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{a} = \left( \frac{1}{2} \mathbf{i} - \frac{1}{2} \mathbf{j} + \frac{3}{2} \mathbf{k} \right) \cdot (\mathbf{i} - \mathbf{j} - \mathbf{k}) = \frac{1}{2} + 1 - \frac{3}{2} = 0 \) and
\[
(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{b} = \left( \frac{1}{2} \mathbf{i} - \frac{1}{2} \mathbf{j} + \frac{3}{2} \mathbf{k} \right) \cdot \left( \frac{1}{2} \mathbf{i} + \frac{1}{2} \mathbf{j} + \frac{1}{2} \mathbf{k} \right) = \frac{1}{4} + 1 + \frac{3}{4} = 1, \text{ so } \mathbf{a} \times \mathbf{b} \text{ is orthogonal to both } \mathbf{a} \text{ and } \mathbf{b}.
\]

10. \( \mathbf{k} \times (\mathbf{i} - 2 \mathbf{j}) = \mathbf{k} \times \mathbf{i} + \mathbf{k} \times (-2 \mathbf{j}) \) by Property 3 of Theorem 11
\[
= \mathbf{k} \times \mathbf{i} + (\mathbf{k} \times -2) (\mathbf{k} \times \mathbf{j}) \text{ by Property 2 of Theorem 11}
\]
\[
= \mathbf{j} + (-2)(-\mathbf{i}) = 2 \mathbf{i} + \mathbf{j} \text{ by the discussion preceding Theorem 11}
\]
28. The parallelogram is determined by the vectors \(\overrightarrow{KL} = (0, 1, 3)\) and \(\overrightarrow{KN} = (2, 5, 0)\), so the area of parallelogram \(KLMN\) is
\[
\left|\overrightarrow{KL} \times \overrightarrow{KN}\right| = \left| \begin{vmatrix} i & j & k \\ 0 & 1 & 3 \\ 2 & 5 & 0 \end{vmatrix} \right| = \left|(-15)i - (-6)j + (-2)k\right| = |-15i + 6j - 2k| = \sqrt{265} \approx 16.28
\]

32.
(a) \(\overrightarrow{PQ} = (1, 2, 1)\) and \(\overrightarrow{PR} = (5, 0, -2)\), so a vector orthogonal to the plane through \(P\), \(Q\), and \(R\) is
\[
\overrightarrow{PQ} \times \overrightarrow{PR} = ((2)(-2) - (1)(0), (1)(5) - (1)(-2), (1)(0) - (2)(5)) = (-4, 7, -10) \ [\text{or any scalar multiple thereof}].
\]
(b) The area of the parallelogram determined by \(\overrightarrow{PQ}\) and \(\overrightarrow{PR}\) is
\[
\left|\overrightarrow{PQ} \times \overrightarrow{PR}\right| = \left|(-4, 7, -10)\right| = \sqrt{16 + 49 + 100} = \sqrt{165},
\]
so the area of triangle \(PQR\) is \(\frac{1}{2} \sqrt{165}\).

36. \(a = \overrightarrow{PQ} = (-4, 2, 4)\), \(b = \overrightarrow{PR} = (2, 1, -2)\) and \(c = \overrightarrow{PS} = (-3, 4, 1)\).
\[
a \cdot (\overrightarrow{b} \times \overrightarrow{c}) = \begin{vmatrix} -4 & 2 & 4 \\ 2 & 1 & -2 \\ -3 & 4 & 1 \end{vmatrix} = -4 \begin{vmatrix} 1 & -2 \\ 4 & 1 \end{vmatrix} - 2 \begin{vmatrix} -2 \\ -3 \end{vmatrix} + 4 \begin{vmatrix} 2 & 1 \\ -3 & 4 \end{vmatrix} = -36 + 8 + 44 = 16,
\]
so the volume of the parallelepiped is 16 cubic units.