Accelerated Multivariable Calculus Fall 2019
Review Problems

These review problems are meant as practice problems for the exam; they will not be collected and graded. Disclaimer: these are just some of the types of problems which might appear on the exam. Just because there is not a problem here on a given topic does not necessarily mean that it will not appear on the exam. Conversely, just because there is a problem here does not mean that there will be a similar problem on the exam.

Stewart: Chapter 12 Review Problems pp. 842–843: 1, 4, 5, 6, 11, 18, 19, 20, 26; Chapter 13 Review Problems pp. 882–883: 6, 8, 17, 19; Chapter 14 Review Problems pp. 982–984: 6, 9, 10, 13, 14, 19, 20, 25, 28, 43, 45, 47. Note: there are answers to the odd numbered problems in the back of the book.

Other suggested problems:

1. Let $\mathbf{v} = (-1, 1, 3)$ and let $\mathbf{w} = (-1, 0, 1)$.

(i) Find a unit vector $\mathbf{u}$ which points in the same direction as $\mathbf{w}$.

(ii) Find the component of $\mathbf{v}$ along $\mathbf{u}$ and the (vector) projection $p_{\mathbf{u}}(\mathbf{v})$.

(iii) Find $\mathbf{v} - p_{\mathbf{u}}(\mathbf{v})$ and verify that it is perpendicular to $\mathbf{u}$.

(iv) Using (iii), compute the distance from $\mathbf{v}$ to the line through the origin in $\mathbb{R}^3$ and $\mathbf{w}$.

(v) Compute the distance from $\mathbf{v}$ to the line through the origin in $\mathbb{R}^3$ and $\mathbf{w}$ by using the fact that the distance is equal to $\|\mathbf{v} \times \mathbf{w}\|/\|\mathbf{w}\|$ and check that it agrees with your answer in (iv).

2. Find the distance from the point $\mathbf{p}$ to the line L given as the set of all vectors of the form $\mathbf{p}_0 + t\mathbf{w}$, where $\mathbf{p} = (1, 2, -2)$, $\mathbf{p}_0 = (0, 0, 1)$, and $\mathbf{w} = (1, 1, -3)$.

3. Find the distance from the point $\mathbf{p} = (2, -1, 3)$ to the plane defined by the equation $2x + 4y - z = 3$.

4. Find the following determinants:

$$\text{(a)}\ \det\begin{pmatrix} 2 & 5 \\ -2 & 7 \end{pmatrix}; \qquad \text{(b)}\ \det\begin{pmatrix} 1 & 7 \\ 1 & 3 \end{pmatrix};$$

$$\text{(c)}\ \det\begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ 1 & -1 & 2 \end{pmatrix}; \qquad \text{(d)}\ \det\begin{pmatrix} 1 & -5 & -5 \\ 2 & -1 & -1 \\ 7 & 1 & 1 \end{pmatrix}.$$

For (a), (b), what is the area of the parallelogram defined by the vectors $\mathbf{0} = (0,0)$ and the two rows $\mathbf{v}_1$, $\mathbf{v}_2$ (i.e. the parallelogram with vertices $\mathbf{0}$, $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_1 + \mathbf{v}_2$)? What is the area of the corresponding triangle (with vertices $\mathbf{0}$, $\mathbf{v}_1$, $\mathbf{v}_2$)?

For (c), (d), what is the volume of the parallelepiped defined by the vectors $\mathbf{0} = (0,0,0)$ and the three rows $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ of the given matrix (i.e. the parallelepiped with vertices $\mathbf{0}$, $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_1 + \mathbf{v}_2$, $\mathbf{v}_1 + \mathbf{v}_3$, $\mathbf{v}_2 + \mathbf{v}_3$, $\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3$)? What is the meaning of your answer for (d)?