Accelerated Multivariable Calculus Fall 2019
Review Problems

These review problems are meant as practice problems for the exam; they will not be collected and graded. Disclaimer: these are just some of the types of problems which might appear on the exam. Just because there is not a problem here on a given topic does not necessarily mean that it will not appear on the exam. Conversely, just because there is a problem here does not mean that there will be a similar problem on the exam.

Stewart: Chapter 12 Review Problems pp. 842–843: 1, 4, 5, 6, 11, 18, 19, 20, 26; Chapter 13 Review Problems pp. 882–883: 6, 8, 17, 19; Chapter 14 Review Problems pp. 982–984: 6, 9, 10, 13, 14, 19, 20, 25, 28, 43, 45, 47. Note: there are answers to the odd numbered problems in the back of the book.

Other suggested problems:

1. Let \( \mathbf{v} = (-1, 1, 3) \) and let \( \mathbf{w} = (-1, 0, 1) \).

   (i) Find a unit vector \( \mathbf{u} \) which points in the same direction as \( \mathbf{w} \).

   (ii) Find the component of \( \mathbf{v} \) along \( \mathbf{u} \) and the (vector) projection \( p_u(\mathbf{v}) \).

   (iii) Find \( \mathbf{v} - p_u(\mathbf{v}) \) and verify that it is perpendicular to \( \mathbf{u} \).

   (iv) Using (iii), compute the distance from \( \mathbf{v} \) to the line through the origin in \( \mathbb{R}^3 \) and \( \mathbf{w} \).

   (v) Compute the distance from \( \mathbf{v} \) to the line through the origin in \( \mathbb{R}^3 \) and \( \mathbf{w} \) by using the fact that the distance is equal to \( \|\mathbf{v} \times \mathbf{w}\|/\|\mathbf{w}\| \) and check that it agrees with your answer in (iv).

2. Find the distance from the point \( \mathbf{p} \) to the line \( L \) given as the set of all vectors of the form \( \mathbf{p}_0 + t\mathbf{w} \), where \( \mathbf{p} = (1, 2, -2) \), \( \mathbf{p}_0 = (0, 0, 1) \), and \( \mathbf{w} = (1, 1, -3) \).

3. Find the distance from the point \( \mathbf{p} = (2, -1, 3) \) to the plane defined by the equation \( 2x + 4y - z = 3 \).
4. Find the following determinants:

(a) \( \det \begin{pmatrix} 2 & 5 \\ -2 & 7 \end{pmatrix} \);  
(b) \( \det \begin{pmatrix} 1 & 7 \\ 1 & 3 \end{pmatrix} \);

(c) \( \det \begin{pmatrix} 2 & 2 & 3 \\ 1 & -1 & 0 \\ 1 & -1 & 2 \end{pmatrix} \);  
(d) \( \det \begin{pmatrix} 1 & -5 & -5 \\ 1 & -1 & 1 \\ 7 & 1 & 1 \end{pmatrix} \).

For (a), (b), what is the area of the parallelogram defined by the vectors \( \mathbf{0} = (0,0) \) and the two rows \( \mathbf{v}_1, \mathbf{v}_2 \) (i.e. the parallelogram with vertices \( \mathbf{0}, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_2 \))? What is the area of the corresponding triangle (with vertices \( \mathbf{0}, \mathbf{v}_1, \mathbf{v}_2 \))?  

For (c), (d), what is the volume of the parallelepiped defined by the vectors \( \mathbf{0} = (0,0,0) \) and the three rows \( \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3 \) of the given matrix (i.e. the parallelepiped with vertices \( \mathbf{0}, \mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_1 + \mathbf{v}_3, \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3 \))? What is the meaning of your answer for (d)?