(1) Hatcher 2.1.9

(2) Hatcher 2.1.11. Use this exercise to conclude that any inclusion \( i : S^k \hookrightarrow S^n, k < n, \) does not have a retract.

(3) Hatcher 2.1.14

(4) Hatcher 2.1.15

(5) a) Hatcher 2.1.20 (assume that \( X \) is a \( \Delta \)-complex and prove this by finding a \( \Delta \)-complex for \( SX \) in terms of a \( \Delta \)-complex for \( X \); or use the long exact sequence)

b) Suppose \( M \) is a \( \Delta \)-complex. Construct a \( \Delta \)-complex on \( \text{Cone}(M) \) and use this to show that \( \tilde{H}_n(\text{Cone}(M)) = 0 \) for all \( n \).

6. Consider a chain complex \( C \) given by \( 0 \rightarrow \mathbb{Z}^k \xrightarrow{\varphi} \mathbb{Z}^n \xrightarrow{\psi} \mathbb{Z}^m \rightarrow 0 \) that is exact; so \( \varphi \) is injective, \( \psi \) is surjective, and \( \text{Image}(\varphi) = \text{Kernel}(\psi) \). Find an chain homotopy between the identity chain map \( \text{Id} : C \rightarrow C \) and the zero chain map \( 0 : C \rightarrow C \).

7. a) Is every chain map \( B_* \rightarrow C_* \) so that \( B_k \rightarrow C_k \) is surjective for all \( k \) induce a surjective map \( H_k(B) \rightarrow H_k(C) \) on homology? If not, give a counterexample.

b) Is every chain map \( B_* \rightarrow C_* \) so that \( B_k \rightarrow C_k \) is injective for all \( k \) induce a injective map \( H_k(B) \rightarrow H_k(C) \) on homology? If not, give a counterexample.

8) Optional:

The simplicial chain complex for a point \( p \) is \( C_0(p) = \mathbb{Z} \) if \( n = 0 \) and \( C_n(p) = 0 \) otherwise; the boundary map is zero for all \( n \). The singular chain complex for \( p \) is \( D_0(p) = \mathbb{Z} \) for all \( n \) (since there is a single, constant map \( \Delta^n \rightarrow p \) for all \( n \)); the boundary map \( \partial_n : D_n(p) \rightarrow D_{n-1}(p) \) is the identity map if \( n \) is even and the zero map if \( n \) is odd. These two chain complexes have the same homology (later we will prove that simplicial and singular homology always agree).

Find chain maps \( f_* : C_* \rightarrow D_* \) and \( g_* : D_* \rightarrow C_* \), and chain homotopies \( h_1 : C_* \rightarrow C_{*+1}, h_2 : D_* \rightarrow D_{*+1} \) so that \( g_* \circ f_* : C_* \rightarrow C_* \) is chain homotopic to the identity chain.
map $Id_* : C_* \to C_*$ (via the chain homotopy $h_1$) and $f_* \circ g_* : D_* \to D_*$ is chain homotopic to the identity chain map $Id_* : D_* \to D_*$ (via the chain homotopy $h_2$).