Problem set 9

Due Wednesday, April 5

1. Show that every line bundle on the projective space $\mathbb{P}^n$ is of the form $\mathcal{O}(m)$ for some m.

2. Show that $\mathcal{O}(m)$ is naturally equivariant for the natural action of $G = GL(n+1)$ on $\mathbb{P}^n$. When can $\mathcal{O}(m)$ be made equivariant for the action of

$$\mathrm{Aut}(\mathbb{P}^n) = PGL(n+1) = GL(n+1)/\text{center} \quad ?$$

3. Compute the cohomology groups of $\mathcal{O}(m)$ from the standard Čech complex. Identify them as representation of G.

4. Construct a minimal G-equivariant resolution of the skyscraper sheaf $\mathcal{O}_0$ of the origin

$$0 \in \mathbb{A}^{n+1}\,.$$

Obtain a relation in $K_G(\mathbb{P}^n)$ by restricting this resolution to $\mathbb{A}^{n+1} \setminus \{0\}$.