Problem set 10

Due Wednesday, April 12

1. Let V be a vector bundle over a algebraic variety X. Globalize problem 4 from HW9, to give a free resolution of the structure sheaf $\iota_* \mathcal{O}_X$ of the zero section

$$\iota : X \hookrightarrow V \,.$$

2. Let

$$\mathbb{P}(V) = (V \setminus \iota(X)) \big/ GL(1)$$

be the bundle of projective spaces associated to V. Use problem 1 to compute the minimal polynomial of the line bundle $\mathcal{O}(1) \in K(\mathbb{P}(V))$ viewed as a module over $K(X)$. (It is a classical theorem, the proof of which we will see later, that $\mathcal{O}(1)$ generates this module).

3. Compute $\iota^* \iota_* \mathcal{O}_X \in K(X)$.

4. Consider the tautological sequence

$$0 \to S = \mathcal{O}(-1) \to \mathbb{C}^n \to Q \to 0$$

over $X = \mathbb{P}(\mathbb{C}^n)$ where S is the tautological subbundle and Q is the tautological quotient bundle of rank $(n-1)$. Let

$$p_1, p_2 : X^2 \to X$$

be the projections onto the two factors. Show that the natural composite map

$$s : p_1^* S \to \mathbb{C}^n \to p_2^* Q$$

where $\mathbb{C}^n$ is the trivial bundle with fiber $\mathbb{C}^n$, induces a section

$$s \in p_1^* S^\vee \otimes p_2^* Q \,,$$

where vee denotes dual, which vanishes precisely over the diagonal in X^2. Deduce that the structure sheaf of the diagonal $\Delta \in X^2$ is in the image of

$$K(X) \otimes K(X) \to K(X^2) \,.$$