1 Chapter 1

1. Naïve set theory, equivalence relations, order relations, functions
2. The natural numbers, the integers, the rational numbers
3. Definition of field, ordered fields
4. The real numbers (thm 1.19 with no proof). The real numbers have the archimedean property. The rational numbers are dense in the reals.
5. Every positive real number has a unique $n$-th root.
6. Definition of the field of the complex numbers and first properties.
7. Schwarz inequality
8. Definition of euclidean space and first properties
9. Exercise 6/7

2 Chapter 2

1. Definition of a metric space.
2. The euclidean space $\mathbb{R}^n$ is a metric space.
3. Neighborhood, limit point, open, closed, bounded, dense
4. Neighborhoods are open
5. Every neighborhood of a limit point contains infinitely many points
6. Finite sets don’t have limit points
7. A metric space is a topological space (thm 2.24)
8. Closure of a set
9. A closed bounded subset of $\mathbb{R}$ contains supremum and infimum.
10. Definition of compact
11. Compacts are closed
12. A closed in a compact is compact
13. The intersection of non-empty nested compact is not empty (thm 2.36+corollary)
14. A k-cell is compact
15. Heine-Borel theorem + Weierstrass theorem

3 Chapter 3

1. Convergent sequences
2. Convergent sequences and limit points (thm 3.2)
3. Basic properties (thm 3.3)
4. Subsequences. A sequence in a compact contains a convergent subsequence. A bounded sequence in a euclidean space also contains a convergent subsequence
5. The set of subsequential limits is closed
6. Upper and lower limit. There is a always a subsequence converging to the upper/lower limit (thm 3.17)
7. Special sequences
8. Cauchy sequences. Convergent implies Cauchy. In a compact space Cauchy implies convergent. In a euclidean space Cauchy implies convergent.
9. Definition of complete space.
10. Ex 24: every metric space can be completed
11. A monotonic sequence converges if and only if it is bounded
12. Series of complex numbers
13. Cauchy criterion for a series
14. If a series converges, then the sequence of the summands converges to zero
15. A non-negative series converges if and only if its partial sums are bounded
16. Comparison of series
17. Geometric series
18. A criterion of Cauchy for the convergence of series whose summands decrease to zero (thm 3.27)
19. Special series
20. Definition of the Euler number. The Euler number is irrational.
21. Absolute convergence
22. Root and ratio test. The root-test is better than the ratio test

23. Power series and radius of convergence

24. Summation by parts. (A technique to study converging series that are not absolutely convergent)

25. Multiplication of series