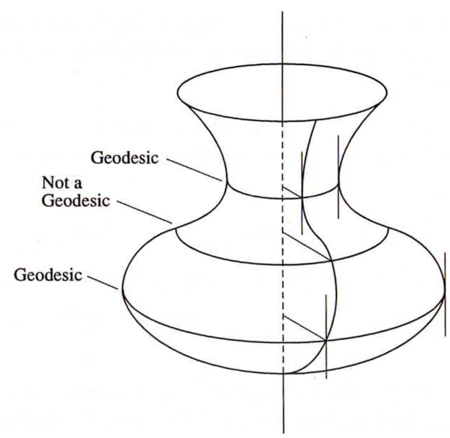
Math 401 HW#8, due Monday 4/4/22 NAME:

Turn in the solutions to #1-#3 in gradescope.

* Exercise §7 (11)
Read through the hint in the book, but don't turn in a solution
Find all the traces of the geodesics joining two given points of a plane, sphere, and right circular cylinder. What can you say about their number? Deduce that, in the case of the unit sphere, a geodesic of length less than π always minimizes the length between its ends.

1. Exercise §7 (12)
If all the geodesics of a connected surface are plane curves, show that the surface is contained either in a plane or in a sphere. *(For full credit, be sure to explain the intermediate calculations and why proportionality between the first and second fundamental forms tells you S is totally umbillical.)*

2. Exercise §7 (13)
Show that each meridian - generating curve - of a surface of revolution is a geodesic. On the other hand, show that a parallel is a geodesic if and only if it is at a critical distance from the axis of revolution. *(Try to do as much of this as you can without reading through the full hint. For full credit, explain how the converse works.)*



3. Exercise §7 (16)
Let $\alpha : I \to S$ be a geodesic in an oriented surface S such that

$$\sigma_{\alpha(t)}(\alpha'(t), \alpha'(t)) = 0$$

for each $t \in I$. Prove that α is a segment of a straight line.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?