Math 4081 HW#8, due Wednesday 4/18/18 **NAME:**

1. (3 points) Let M be a smooth manifold and $\alpha \in \Omega^1(M)$. Show that a smooth distribution $\xi = \ker \alpha$ is maximally (e.g. nowhere) nonintegrable whenever $d\alpha|_\xi$ is nondegenerate. *Hint: Use HW #6.3.*

2. (4 points) Show that a smooth distribution $D \subset TM$ is involutive if and only if $\mathscr{I}(D)$ is a differential ideal.

3. (4 points) For $k > 0$, define a map K from k-forms on $\mathbb{R}^n$ to $(k-1)$-forms on $\mathbb{R}^n$ as follows. If α is a k-form, write $\alpha = dx^1 \wedge \beta_I dx^I + \gamma_J dx^J$ where the multi-indices I and J do not include 1. Define

$$K\alpha(x^1, \ldots, x^n) = \left(\int_0^{x^1} \beta_I(t, x^2, \ldots, x^n) dt \right) dx^I.$$

Let V denote the subspace $(x^1 = 0) \subset \mathbb{R}^n$, let $\imath : V \to \mathbb{R}^n$ denote the inclusion map, and let $\pi : \mathbb{R}^n \to V$ denote the projection $(x^1, \ldots, x^n) \mapsto (0, x^2, \ldots, x^n)$.

(a) Show that

$$d \circ K + K \circ d = 1 - \pi^* \circ \imath^*.$$

(b) Use the above result and induction on n to show that if $k > 0$ then every closed k-form on $\mathbb{R}^n$ is exact.

4. (3 points) Lee 16-2 SECOND
Let $T^2 = S^1 \times S^1 \subset \mathbb{R}^4$ denote the 2-torus, defined as the set of points (w, x, y, z) such that $w^2 + x^2 + y^2 + z^2 = 1$, with the product orientation determined by the standard orientation on S^1 (e.g. don't worry about it). Compute $\int_{T^2} \omega$, where ω is the following 2-form on $\mathbb{R}^4$:

$$\omega = xyz \, dw \wedge dy.$$

5. How difficult was this assignment?