Math 401 HW#2, due Monday 1/24/22 **NAME:**

Do not upload solutions to the **In Class Warm Up Problems** to gradescope.
UPDATED: Exercise about parametrization giving a diffeo is moved to HW 3.

⋆ **Multi Warm Up**
Find an equation for the tangent plane to the following parametrized surface at the point $(1, -2, 1)$.

$$S = \begin{cases} x &= e^{u-v} \\ y &= u - 3v \\ z &= \frac{1}{2}(u^2 + v^2) \end{cases}$$

⋆ **Multi Warm Up**
Find a parametrization for each of the following surfaces (perhaps involving an angular variable that is defined only up to multiples of 2π).

(a) The surface obtained by revolving the curve $z = f(x)$, $a < x < b$ in the xz-plane around the z-axis, where $a > 0$.

(b) The surface obtained by revolving the curve $z = f(x)$, $a < x < b$ in the xz-plane around the x-axis, where $f(x) > 0$.

(c) The lower sheet of the hyperboloid $z^2 - 2x^2 - y^2 = 1$.

(d) The cylinder $x^2 + z^2 = 9$.

1. **For each of the following maps $f : \mathbb{R}^2 \to \mathbb{R}^3$,**
   - Describe or sketch the (possibly singular) surface $S = f(\mathbb{R}^2)$
   - Find a description of S as the locus of an equation $F(x, y, z) = 0$.
   - Find the points where $\partial_u f$ and $\partial_v f$ are linearly dependent, and describe the singularities of S (if any) at these points.

   (a) $f(u, v) = (2u + v, u - v, 3v)$

   (b) $f(u, v) = (au \cos v, bu \sin v, u)$ with $a, b > 0$

   (c) $f(u, v) = (u \cos v, u \sin v, u^2)$

2. **Exercise 2.24**
   Prove that one-sheeted cone is not a surface: $C = \{(x, y, z) \in \mathbb{R}^3 \mid z^2 = x^2 + y^2, z \geq 0\}$

3. **First part of Exercise 2 (7)**
   Let $S = \{(x, y, z) \in \mathbb{R}^3 \mid e^{x^2} + e^{y^2} + e^{z^2} = a\}$, with $a > 3$. Prove that S is a surface.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?