Math 401 HW#2, due Wednesday 9/11/19 NAME:

1. Multi Warm Up
   Let C be the circle formed by intersecting the plane $x+z=1$ with the sphere $x^2+y^2+z^2=1$.
   (a) Find a parametrization of C.
   (b) Find parametric equations for the tangent line to C at the point $\left(\frac{1}{2}, -\frac{1}{\sqrt{2}}, \frac{1}{2}\right)$

2. Multi Warm Up
   Find an equation for the tangent plane to the following parametrized surface at the point $(1,-2,1)$.
$$S = \begin{cases} x &= e^{u-v} \\ y &= u-3v \\ z &= \frac{1}{2}(u^2+v^2) \end{cases}$$

3. Multi Warm Up
   Find a parametrization for each of the following surfaces (perhaps involving an angular variable that is defined only up to multiples of 2π).
   (a) The surface obtained by revolving the curve $z=f(x)$, $a<x<b$ in the xz-plane around the z-axis, where $a>0$.
   (b) The surface obtained by revolving the curve $z=f(x)$, $a<x<b$ in the xz-plane around the x-axis, where $f(x)>0$.
   (c) The lower sheet of the hyperboloid $z^2-2x^2-y^2=1$.
   (d) The cylinder $x^2+z^2=9$.

4. Multi Warm Up
   For each of the following maps $f:\mathbb{R}^2\to\mathbb{R}^3$, describe the (possibly singular) surface $S=f(\mathbb{R}^2)$ and find a description of S as the locus of an equation $F(x,y,z)=0$. Find the points where $\partial_u f$ and $\partial_v f$ are linearly dependent, and describe the singularities of S (if any) at these points.
   (a) $f(u,v)=(2u+v, u-v, 3v)$
   (b) $f(u,v)=(au\cos v, bu\sin v, u)$ with $a,b>0$
   (c) $f(u,v)=(u\cos v, u\sin v, u^2)$

5. Exercise 2.24
   Prove that below one-sheeted is not a surface:
$$C=\{(x,y,z)\in\mathbb{R}^3 \mid z^2=x^2+y^2, z\geq 0\}$$

6. Exercise 2 (7)
   Let
$$S=\{(x,y,z)\in\mathbb{R}^3 \mid e^{x^2}+e^{y^2}+e^{z^2}=a\}$$
   with $a>3$. Prove that S is a surface.

* Assignment Reflections
   How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?