Math 4081 HW#2, due Wednesday 2/7/18 NAME:

1. Lee 3-4 [SECOND] = 4-1 [FIRST].
   Show that TS^1 is diffeomorphic to $S^1 \times \mathbb{R}$.

2. Show that if M and N are smooth manifolds and if $p \in M$ and $q \in N$, then there is a canonical isomorphism

$$T_{(p,q)}(M \times N) = T_pM \oplus T_qN.$$

Describe this isomorphism in terms of (a) derivations and (b) linear combinations of partial derivatives with respect to coordinate charts.

3. Lee 8-10 [SECOND]
   Let M be the open submanifold of $\mathbb{R}^2$ where both x and y are positive and let $F : M \to N$ be the map

$$F(x, y) = \left(xy, \frac{y}{x}\right).$$

   Show that F is a diffeomorphism, and compute $F_* X$ and $F_* Y$ where

$$X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}; \qquad Y = y\frac{\partial}{\partial x}$$

4. Lee 8-11 [SECOND] = 4-5 [FIRST]
For each of the following vector fields on the plane, compute its coordinate representation in polar coordinates on the right half-plane $\{(x, y) \in \mathbb{R}^2 \mid x > 0\}$.

$$
\begin{aligned}
\text{(a)}\quad X &= x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y} \\
\text{(b)}\quad Y &= x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} \\
\text{(c)}\quad Z &= (x^2 + y^2)\,\frac{\partial}{\partial x}
\end{aligned}
$$

5. Lee 8-16 [SECOND] = 4-11 [FIRST]

For each of the following pairs of vector fields $X,\ Y$ defined on $\mathbb{R}^3$, compute the Lie bracket $[X, Y]$.

$$
\begin{aligned}
&\text{(a)}\quad X_1 = y\frac{\partial}{\partial z} - 2xy^2\frac{\partial}{\partial y}; && Y_1 = \frac{\partial}{\partial y} \\
&\text{(b)}\quad X_2 = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x} && Y_2 = y\frac{\partial}{\partial z} - z\frac{\partial}{\partial y} \\
&\text{(c)}\quad X_3 = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}; && Y_3 = x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}
\end{aligned}
$$

6. How difficult was this assignment?