Math 401 HW#1, due Wednesday 9/4/19 NAME:

1. Multi Warm Up
   Reparametrize the below curve by arc length from $(1,1,0)$ in the direction of increasing t
   $$r(t) = \langle e^t, e^{-t}, t\sqrt{2}\rangle.$$
   Then compute $\mathbf{T}, \mathbf{N}, \mathbf{B}$, $k(s)$, and $\tau(s)$.

2. Exercise 1.8
   Let $\alpha : I \to \mathbb{R}^3$ be a curve and let $M : R^3 \to R^3$ be a rigid motion. Prove that
   $$L_b^a(\alpha) = L_b^a(M \circ \alpha).$$
   That is, rigid motions preserve the length of curves.

3. Exercise 1.9
   Let $\alpha : I \to \mathbb{R}^3$ be a curve and $[a,b] \subset I$. Prove that
   $$|\alpha(a) - \alpha(b)| \leq L_b^a(\alpha).$$
   In other words, straight lines are the shortest curves joining two given points.

4. Exercise 1.12
   Let $\phi : J \to I$ be a diffeomorphism and let $\alpha : I \to \mathbb{R}^3$ be a curve. Given $[a,b] \subset J$ with $\phi([a,b]) = [c,d]$ prove that $L_a^b(\alpha \circ \phi) = L_c^d(\alpha)$.

5. Exercise 1.13
   Consider the logarithmic spiral $\alpha : \mathbb{R} \to \mathbb{R}^2$ given by
   $$\alpha(t) = (ae^{bt}\cos t, ae^{bt}\sin t)$$
   with $a > 0,\ b < 0$. Compute the arc length function $S : \mathbb{R} \to \mathbb{R}$ for t_0 corresponding to $t_0 \in \mathbb{R}$. Reparametrize this curve by arc length and describe/sketch its trace.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience?