Math 451 HW # 9, due Friday 11/11/22 NAME:

1. Lee 14-6 [Second Edition]
   Define a 2-form ω on $\mathbb{R}^3$ by
   $$\omega = xdy \wedge dz + ydz \wedge dx + zdx \wedge dy.$$
   (a) Compute ω in spherical coordinates (ρ, φ, θ) defined by
   $$(x, y, z) = (\rho \sin \varphi \cos \theta, \rho \sin \varphi \sin \theta, \rho \cos \varphi).$$
   (b) Compute $d\omega$ in both Cartesian and spherical coordinates and verify that both expressions represent the same 3-form.
   (c) Compute the pullback $\iota^*_{S^2}\omega$ to S^2, using coordinates (φ, θ) on the open subset where these coordinates are defined.
   (d) Show that $\iota^*_{S^2}\omega$ is nowhere zero.

2. Lee 14-7 [Second Edition]
   Let $M = \mathbb{R}^2$ and $N = \mathbb{R}^3$, $\omega = ydz \wedge dx$, and $F : M \to N$ is the smooth map defined by
   $$F(\theta, \varphi) = ((\cos \varphi + 2) \cos \theta, (\cos \varphi + 2) \sin \theta, \sin \varphi) .$$
   Compute $d\omega$ and $F^*\omega$. *Observe that it is possible to verify by direct computation that* $F^*(d\omega) = d(F^*\omega)$*, but do not do so.*

3. Check in local coordinates that if α is a 1-form and V and W are vector fields on M, then
   $$d\alpha(V, W) = V\alpha(W) - W\alpha(V) - \alpha([V, W]).$$

4. Lee 19-1 [Second Edition]
   Show that a smooth distribution $D \subset TM$ is involutive if and only if $\mathcal{I}(D)$ is a differential ideal.
   ($\Rightarrow$*): Use Prop 14.32. (*$\Leftarrow$*): Use Theorem 19.7.*

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?