**Math 451 HW # 5, due Friday 9/30/22** **NAME:**

1. Lee 8-10 [Second Edition]
Let $M$ be the open submanifold of $\mathbb{R}^2$ where both $x$ and $y$ are positive and let $F : M \to M$ be the map
$$F(x,y) = \left(xy, \frac{y}{x}\right).$$
Show that $F$ is a diffeomorphism, and compute $F_*X$ and $F_*Y$ where
$$X = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}; \qquad Y = y\frac{\partial}{\partial x}$$
Note: The definition of the pushforward yields $(F_*Z)_{(s,t)} = dF_{F^{-1}(s,t)}Z_{F^{-1}(s,t)}$.

2. Lee 8-11 [Second Edition]
For each of the following vector fields on the plane, compute its coordinate representation in polar coordinates on the right half-plane $\{(x,y) \in \mathbb{R}^2 \mid x > 0\}$.
$$\text{(b)} \quad X = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}$$
$$\text{(a)} \quad Y = x\frac{\partial}{\partial x} + y\frac{\partial}{\partial y}$$

3. Lee 9-3 [Second Edition]
Compute the flow of each of the following vector fields on $\mathbb{R}^2$:
   (a) $X = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y}$
   (b) $Y = x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}$

4. Lee [Second Edition]
Define vector fields $X$ and $Y$ on the plane as in (1) by
$$X = x\frac{\partial}{\partial x} - y\frac{\partial}{\partial y} \qquad Y = x\frac{\partial}{\partial y} + y\frac{\partial}{\partial x}$$
Recall that you just computed the flows $\phi$ and $\psi$ of $X$ and $Y$ in the previous problem. Now verify that the flows do not commute by finding explicit open intervals $I$ and $J$ containing 0 such that $\phi_s \circ \psi_t$ and $\psi_t \circ \phi_s$ are both defined for all $(s,t) \in I \times J$, but they are unequal for some such $(s,t)$.

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?