Math 451 HW # 4, due Friday 9/23/22 **NAME:**

1. (a) Suppose that $A : \mathbb{R}^k \to \mathbb{R}^n$ is a linear map and V is a vector subspace of $\mathbb{R}^n$. Check that $A \pitchfork V$ is equivalent to $A(\mathbb{R}^k) + V = \mathbb{R}^n$.
   (b) If V and W are linear subspaces of $\mathbb{R}^n$, check that $V \pitchfork W$ is equivalent to $V + W = \mathbb{R}^n$.
2. For which values of R does the hyperboloid defined by $x^2 + y^2 - z^2 = 1$ intersect the sphere $x^2 + y^2 + z^2 = R$ transversely? What does the intersection look like for different values of R?
3. (Lee Second 6-9) Let $F : \mathbb{R}^2 \to \mathbb{R}^3$ be the map defined by

$$F(x, y) = (e^y \cos x, e^y \sin x, e^{-y}).$$

   (a) For which positive numbers r is F transverse to the 2-sphere of radius r, $S_r(0) \subset \mathbb{R}^3$?
   (b) For which positive numbers r is $F^{-1}(S_r(0))$ an embedded submanifold of $\mathbb{R}^2$?

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?