Math 451 HW # 3, due Friday 9/16/22 **NAME:**

1. Lee 3-6 [Second Edition]
   Consider $\mathbb{S}^3$ as the unit sphere in $\mathbb{C}^2$ under the usual identification $\mathbb{C}^2 \leftrightarrow \mathbb{R}^4$. For each $z = (z^1, z^2) \in \mathbb{S}^3$, define a curve $\gamma_z : \mathbb{R} \to \mathbb{S}^3$ by $\gamma_z(t) = (e^{it}z^1, e^{it}z^2)$. Show that γ_z is a smooth curve whose velocity is never zero.

2. Lee 4-6 [Second Edition]
   Let M be a nonempty smooth compact manifold. Show that there is no smooth submersion $F : M \to \mathbb{R}^k$ for any $k > 0$. A submersion is a smooth map whose differential is surjective.

3. Lee 5-1 [Second Edition]
   Consider the map $\Phi : \mathbb{R}^4 \to \mathbb{R}^4$ defined by

   $$\Phi(x, y, s, t) = (x^2 + y, x^2 + y^2 + s^2 + t^2 + y).$$

   Show that $(0, 1)$ is a regular value of Φ in the sense of multivariable calculus. *(It turns out that the level set $\Phi^{-1}(0, 1)$ is diffeomorphic to S^2, but this is surprisingly painful to prove, so I won't ask you to do this.)*

4. Lee 5-10 [Second Edition]
   For each $a \in \mathbb{R}$, let M_a be the subset of $\mathbb{R}^2$ defined by

   $$M_a = \{(x, y) \mid y^2 = x(x - 1)(x - a)\}.$$

   For which values of a is M_a an embedded submanifold of $\mathbb{R}^2$? *For which values can M_a be given a topology and smooth structure making it into an immersed submanifold? You don't need to give a rigorous proof for the a values which don't produce embedded submanifolds; this is surprisingly painful.*

* Which problems provided a worthwhile learning experience? How many hours did you spend on it?