Math 222 in class problems **Week: March 22, 2021** **Name:**

1. Let C be the circle formed by intersecting the plane $x + z = 1$ with the sphere $x^2 + y^2 + z^2 = 1$. Find a parametrization of C.

2. Find the arclength of $\mathbf{r}(t) = (\ln t, 2t, t^2)$, $t \in [1, e]$. (Folland: e^2)

3. Compute $\int_C \sqrt{z}ds$, where C is parametrized by $\mathbf{r}(t) = (2\cos t, 2\sin t, t^2)$, $0 \le t \le 2\pi$. (Folland: $\frac{2}{3}[(1+4\pi^2)^{3/2} - 1]$)

4. Find the work done by the vector field $F(x, y, z) = \langle y^2, 2xy + e^{3z}, 3ye^{3z}\rangle$ along the line segment starting at $(0, 0, 1)$ and ending at $(2, 1, 0)$

5. On HW 7: Jones 12.E, 12-7
   Define a 1-form α on the punctured plane $\mathbb{R}^2 \setminus \{0\}$ by
$$\alpha = \left(\frac{-y}{x^2 + y^2}\right) dx + \left(\frac{x}{x^2 + y^2}\right) dy.$$
   (a) Calculate $\int_C \alpha$ for any circle C of radius R around the origin.
   (b) Prove that in the half plane $\{x > 0\}$, α is the differential of a function.

6. Match the equations of vector fields with their graphs. Determine which vector fields in 11-18 are conservative, and for the ones which are, find their potential functions.

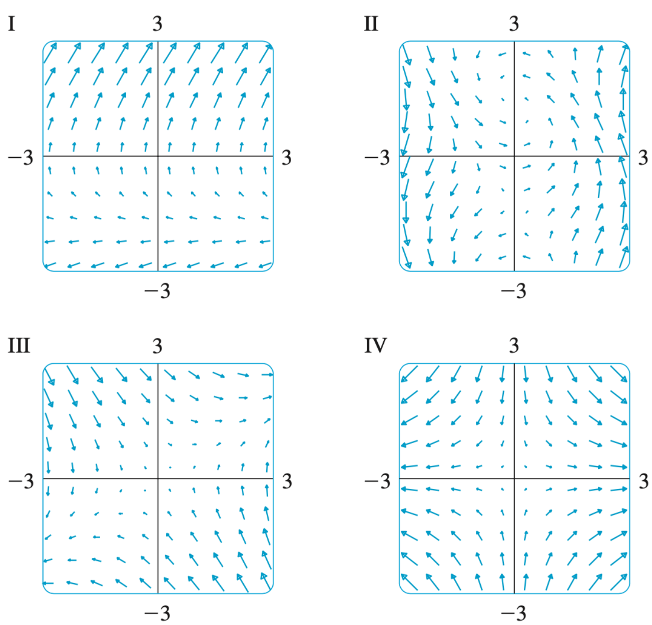
11–14 Match the vector fields **F** with the plots labeled I–IV. Give reasons for your choices.

11. $\mathbf{F}(x, y) = \langle x, -y\rangle$

12. $\mathbf{F}(x, y) = \langle y, x - y\rangle$

13. $\mathbf{F}(x, y) = \langle y, y + 2\rangle$

14. $\mathbf{F}(x, y) = \langle \cos(x + y), x\rangle$



15–18 Match the vector fields **F** on $\mathbb{R}^3$ with the plots labeled I–IV. Give reasons for your choices.

15. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$

16. $\mathbf{F}(x, y, z) = \mathbf{i} + 2\,\mathbf{j} + z\,\mathbf{k}$

17. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + 3\,\mathbf{k}$

18. $\mathbf{F}(x, y, z) = x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k}$