**Math 222 Midterm, due Monday 2/15/21 at 11pm CST** **NAME:**

This is a 90 minute open notes exam, but you can take 2 hours to do it. Pick 5 of the following 6 problems to do. You may use any of the listed course materials (Jones, Folland, or Hutchings videos) but are not permitted to use the internet or material beyond a sheet of trig identities, your favorite unit circle, course textbooks, Prof Jo's lecture notes, in class problem solutions, and homework solutions. You are not permitted to speak with or email/text anyone about this midterm except Prof Jo. Write and sign the Rice honor pledge (and upload to gradescope).

1. Problem 7-10: Jones
   For which dimensions do both frames $\{\varphi_1, \ldots, \varphi_n\}$ and $\{\varphi_n, \varphi_1, \varphi_2, \ldots, \varphi_{n-1}\}$ for $\mathbb{R}^n$ have the same orientation?

2. Rotations problem (Jones lite)
   Let
   $$A = \begin{pmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$$
   Find $\hat{w}$ and $\theta$ such that $A = R(\hat{w}, \theta)$.
   *WARNING: There is no clever trick to get $\hat{w} = \mathbf{k}$ and $\theta = \pi$. (Optional: Why?)*

3. Folland Integration Proof
   Prove that if $f$ is integrable on a bounded rectangle $R$ and $c \in \mathbb{R}$, then $cf$ is integrable on $R$, and
   $$\iint_R cf dA = c \iint_R f dA$$
   Suggestions: First show that if $c \geq 0$, then $s_P(cf) = cs_P(f)$ and $S_P(cf) = cS_P f$ while if $c < 0$, then $s_P(cf) = cS_P(f)$ and $S_P(cf) = cs_P f$.

4. Folland Polar Coordinates
   (a) Let $a > 0$. Prove that a circle based at $(a, 0)$ of radius $a$ can be expressed as $r = 2a\cos\theta$.
   (b) Find the volume of the region above the $xy$-plane, below the cone $z = 2 - \sqrt{x^2 + y^2}$ and inside the cylinder $(x-1)^2 + y^2 = 1$.

   Notes: *Be careful with your $\theta$ bounds! Recall* $\cos^3\theta = (\cos^2\theta)\cos\theta = (1 - \sin^2\theta)\cos\theta$, $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

5. Folland Reverse Integration
   Express each of the following iterated integrals as a double integral and as an iterated integral in the opposite order. (That is, find the region of integration for the double integral and the limits of integration for the other iterated integral. You may need more than one iterated integral to express the reversed limits of integration.)
   (a) $\displaystyle\int_1^e \int_0^{\ln x} f(x, y)\, dy\, dx$
   (b) $\displaystyle\int_{-2}^3 \int_{-y}^{6-y^2} f(x, y)\, dx\, dy$

6. Stewart Triple Integral Limits
   Find the limits of integration (include a labelled graph and a short explanation of how you derived them), but DO NOT EVALUATE the triple integral
   $$\iiint_T xz\, dx\, dy\, dz$$
   where $T$ is the solid tetrahedron with vertices $(0, 0, 0)$, $(1, 0, 1)$, $(0, 1, 1)$, $(0, 0, 1)$.

* Midterm Reflections
  How difficult was this midterm? How long did it take you? Does the pace of class feel too fast, too slow, or reasonable? How is the balance of theory with computations? What has been your favorite topic so far?