Math 222 HW#9, due Thursday 4/22/21 **NAME:**

Choose 5 out of 6 problems. This will be a longer homework set to complete but hopefully still reasonable.

1. Guided Jones 11-10,  d'oh
   Suppose $0 \le a \le b$. Find the surface area of the torus obtained by revolving the circle $(x-b)^2+z^2 = a^2$ in the xz-plane about the z-axis.
   Suggestion: Show that the torus admits the parametrization $0 \le \varphi, \theta \le 2\pi$ by

$$\begin{aligned} x &= (b + a\cos\varphi)\cos\theta \\ y &= (b + a\cos\varphi)\sin\theta \\ z &= a\sin\varphi \end{aligned}$$

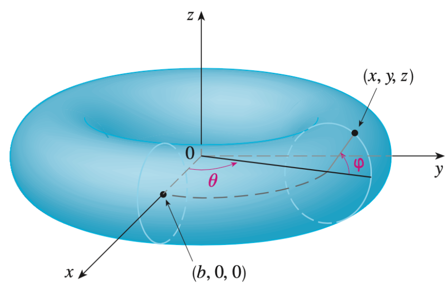

Figure 1: The hollow blue donut

2. Use Stoke's theorem to evaluate $\displaystyle\int_C y\ dx + y^2\ dy + (x + 2z)\ dz$, where C is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and the plane $y + z = a$, oriented counterclockwise as viewed from above. (Remember, it was no fun to parametrize this circle a few weeks ago in class. Setting up a surface integral will be very reasonable. The answer is $-\pi a^2/\sqrt{2}$.)

3. Let $F(x, y, z) = \langle 2x, 2y, x^2 + y^2 + z^2\rangle$ and let S be the lower half of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{27} = 1$. Use Stokes' Theorem to calculate the flux of the curl F across S from the lower side to the upper side. (You definitely don't want to set up a surface integral, but a line integral is pretty reasonable. You'll get 0 as your answer.)

4. Compare with Jones Problem 13-3
   Define the vector field F on the complement of the z-axis by $F(x, y, z) = \dfrac{-y\mathbf{i} + x\mathbf{j}}{x^2 + y^2}$
   (a) Show that curl$F$=0
   (b) Show by a direct calculation that $\int_C F \cdot d\mathbf{x} = 2\pi$ for any horizontal circle C centered at a point on the z-axis.
   (c) Why do (a) and (b) not contradict Stokes' theorem?

5. If S is a sphere and F satisfies the hypotheses of Stokes' Theorem, show that $\displaystyle\iint_S \text{curl}F \cdot \mathbf{n}\ dA = 0$. You need to do more than state that S is a closed manifold, hint: what happens with respect to orientation of the boundary when you split a sphere in half and have to use an outward normal VF?

6. Let S be a smooth oriented surface in $\mathbb{R}^3$ with piecewise smooth, compatibly oriented boundary ∂S. Suppose that f and g are C^1 functions on some open set containing S. Show that

$$\int_{\partial S} f\nabla g \cdot d\mathbf{x} = \iint_S (\nabla f \times \nabla g) \cdot \mathbf{n}\ dS$$

* Assignment Reflections
  How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?