Math 222 HW#6, due Thursday 3/25/21 **NAME:**

1. Folland 3.4, Jones 10.G & 10.I
   Let $(u, v) = F(x, y) = (x - y, xy)$.

   (a) Sketch some of the curves $x - y =$ constant and $xy =$ constant in the xy-plane. Which regions in the xy-plane map onto the rectangle in the uv-plane given by $0 \le u \le 1,\ 1 \le v \le 4$? There are two of them; draw a picture of them.

   (b) Compute the derivative DF and the Jacobian $J = \det DF$

   (c) The Jacobian J vanishes at (a, b) precisely when the gradients $\nabla u(a, b)$ and $\nabla v(a, b)$ are linearly dependent, i.e., when the level sets of u and v passing through a and b are tangent to each other. Use your sketch of the level sets in (a) to show pictorially that this assertion is correct.

   (d) Notice that $F(2, -3) = (5, -6)$. Compute explicitly the local inverse G of F such that $G(5, -6) = (2, -3)$ and compute its derivative DG.

   (e) Show by explicit calculation that the matrices $DF(2, -3)$ and $DG(5, -6)$ are inverses of each other.

2. Folland 3.4,
   Let $F : \mathbb{R}^3 \to \mathbb{R}^3$ be the spherical coordinate mapping:

   $$(x, y, z) = F(\rho, \varphi, \theta) = (\rho \sin\varphi \cos\theta, \rho \sin\varphi \sin\theta, \rho \cos\varphi).$$

   (a) Describe (or draw) the surfaces in xyz-space that are the images of the planes $\rho =$ positive constant, $\varphi =$ constant (check $(0, \frac{\pi}{2})$, $\frac{\pi}{2}$, $(\frac{\pi}{2}, \pi)$ separately), and $\theta =$ constant (in $[0, 2\pi)$).

   (b) Compute the derivative DF and show the Jacobian is $\det DF(\rho, \varphi, \theta) = \rho^2 \sin\varphi$.

   (c) What is the condition on the point $(\rho_0, \varphi_0, \theta_0)$ for F to be locally invertible about this point? What is the corresponding condition on $(x_0, y_0, z_0) = F(\rho_0, \varphi_0, \theta_0)$?

3. Folland, Jones 10.F
   Calculate $\iint_S (x + y)^4 (x - y)^{-5} dA$ where S is the square $-1 \le x + y \le 1$, $1 \le x - y \le 3$.

4. Stewart (on in class problems)
   Evaluate the integral by making an appropriate change of variables:

   $$\iint_R \sin(9x^2 + 4y^2) dA,$$

   where R is the region in the first quadrant bounded by the ellipse $9x^2 + 4y^2 = 1$.

* Assignment Reflections
How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?