Math 222 HW#5, due Thursday 3/18/21 **NAME:**

If you elect to do the Folland proof (#1), you may omit solutions to #3 & #7 and receive full credit. Alternatively, you may elect to submit solutions to #2-7 and omit the proof of #1 to receive full credit.

1. Folland 4.2: Zero Content Proof. Let $f : [a, b] \to \mathbb{R}$ be an integrable function.

   (a) Show that the graph of f in $\mathbb{R}^2$ has zero content.

   Suggestion: Given a partition P of $[a, b]$, interpret $s_P f - S_P f$ as a sum of areas of rectangles that cover the graph of f.

   (b) Suppose $f \geq 0$, and let $S = \{(x, y) : x \in [a, b],\ 0 \leq y \leq f(x)\}$. Show that S is measurable, e.g. S is bounded and and its boundary ∂S has zero content in $\mathbb{R}^2$, and that its area (which we more precisely defined in the zero content and theory handout) equals $\int_a^b f(x)dx$.

2. Stewart
   Sketch the solid whose volume is given by the integral but DO NOT EVALUATE the integral.

   (a) $\displaystyle\int_{-\pi/2}^{\pi/2}\int_0^2\int_0^{r^2} r\ dz\ dr\ d\theta$

   (b) $\displaystyle\int_0^2\int_0^{2\pi}\int_0^r r\ dz\ d\theta\ dr$

3. Stewart
   Evaluate the integral by changing to cylindrical coordinates. Sketch the region of integration.

   (a) $\displaystyle\int_{-2}^2\int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}}\int_{\sqrt{x^2+y^2}}^2 xz\ dz\ dx\ dy$

   (b) $\displaystyle\int_0^3\int_0^{\sqrt{9-x^2}}\int_0^{9-x^2-y^2}\sqrt{x^2+y^2}\ dz\ dy\ dx$

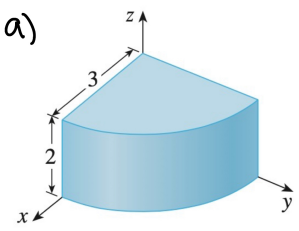
4. Stewart
   Sketch the solid whose volume is given by the integral but DO NOT EVALUATE the integral.

   (a) $\displaystyle\int_0^{\pi/6}\int_0^{\pi/2}\int_0^3 \rho^2\sin\varphi\ d\rho\ d\theta\ d\varphi$

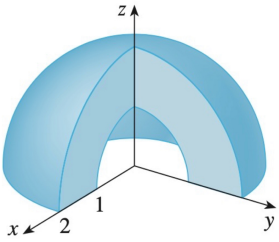
   (b) $\displaystyle\int_0^{\pi/4}\int_0^{2\pi}\int_0^{\sec\varphi} \rho^2\sin\varphi\ d\rho\ d\theta\ d\varphi$

5. Stewart
   Set up the triple integral of an arbitrary continuous function $f(x, y, z)$ in cylindrical or spherical coordinates over the solid region shown.

a)



b)



6. Folland, Jones
   Find the volume of the sphere $x^2 + y^2 + z^2 = 4$ lying above the plane $z = 1$.

7. Folland, Jones
   Find the volume of the region inside both the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 = 1$.

* Assignment Reflections
   How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?