Math 222 HW#1, due Thursday 2/4/21 NAME:

1. Problem 7-1: Jones
   Let $x \in \mathbb{R}^3$ be thought of as a fixed vector. Then $x \times y$ is a linear function from $\mathbb{R}^3$ to $\mathbb{R}^3$ and thus can be represented in a unique way as a matrix times the column vector y. Show that in fact
   $$x \times y = \begin{pmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{pmatrix} y.$$

2. Problem 7-2: Jones
   Assuming $x \neq 0$ in the preceding problem, find the characteristic polynomial of the 3×3 matrix given there. What are its eigenvalues?

3. §12.4 Stewart
   Find the area of the parallelogram with vertices $P(1,0,2)$, $Q(3,3,3)$, $R(7,5,8)$, and $S(5,2,7)$.

4. §12.4 Stewart
   Find the volume of the parallelepiped determined by the vectors
   $$\mathbf{a} = \hat{\imath} + \hat{\jmath}, \quad \mathbf{b} = \hat{\jmath} + \hat{k}, \quad \mathbf{c} = \hat{\imath} + \hat{\jmath} + \hat{k}$$

5. §12.4 Stewart
   (a) Find a nonzero vector orthogonal to the plane through the points P, Q, and R;
   (b) Find the area of the triangle PQR:
   $$P(0,0,-3), \quad Q(4,2,0), \quad R(3,3,1)$$

6. §12.4 Stewart
   If $\mathbf{a} \cdot \mathbf{b} = \sqrt{3}$ and $\mathbf{a} \times \mathbf{b} = \langle 1,2,2 \rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.

7. §12.4 Stewart
   Suppose that $\mathbf{a} \neq 0$. Prove or work out a counter example to each of the following:
   (a) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?
   (b) If $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?
   (c) If $\mathbf{a} \cdot \mathbf{b} = \mathbf{a} \cdot \mathbf{c}$ and $\mathbf{a} \times \mathbf{b} = \mathbf{a} \times \mathbf{c}$, does it follow that $\mathbf{b} = \mathbf{c}$?

8. §12.4 Stewart
   (a) Let P be a point not on the line L that passes through the points Q and R in $\mathbb{R}^3$. Show that the distance d from the point P to the line L is
   $$d = \frac{|\mathbf{a} \times \mathbf{b}|}{|\mathbf{a}|},$$
   where $\mathbf{a} = \overrightarrow{QR}$ and $\mathbf{b} = \overrightarrow{QP}$.
   (b) Use the formula in part (a) to find the distance from the point $P(1,1,1)$ to the line through $Q(0,6,8)$ and $R(-1,4,7)$.

* Assignment Reflections (optional)
  How difficult was this assignment? How many hours did you spend on it? Which problems did you find to provide a worthwhile learning experience? Should I be assigning a similar number of problems, fewer problems, or more problems in the future? Is there a good mix of theory and computations?