Problem 1. Consider the ellipse

\((E) \frac{x^2}{4} + \frac{y^2}{9} = 1.\)

Clearly (E) intersects the x-axis at \(A(2, 0)\) and \(B(-2, 0)\) and the y-axis at \(C(0, 3)\) and \(D(0, -3)\). In the lecture, by using our intuitive understanding of the curvature, we guessed that (E) should have the maximal curvature at C and D and (E) should have the minimal curvature at A and B. This problem is to confirm our intuition. To ease the discussion, let us parametrize (E) by

\[x = 2\cos(t), \quad y = 3\sin(t) \quad (0 \leq t \leq 2\pi).\]

a. Calculate the curvature \(\kappa(t)\) of (E) at the point \(P(2\cos(t), 3\sin(t))\).

b. Find the values of \(t\) such that \(\kappa(t)\) attains its maximal and minimal values.

Problem 2. A circle of radius \(R\) and centered at \(O(0,0)\) moves by curvature if at the time \(t\), its radius \(R(t)\) changes with a rate \(-\frac{1}{R(t)}\):

\[R'(t) = -\frac{1}{R(t)}.\]

Recall that \(-\frac{1}{R(t)}\) is the curvature of the circle of radius \(R(t)\). The minus sign in the rate of change of \(R(t)\) indicates that radius of the circle is decreasing. In other words, the circle moves towards its center with the speed being equal to its curvature. This kind of motion (especially in higher dimensions) has many applications, for instance, in option pricing, motion of grains in annealing metals, and crystal growth.

At the time \(t = 0\), the radius of the circle is \(R\). Thus \(R(0) = R\).

a. Compute explicitly the radius \(R(t)\) at the time \(t\) in terms of \(R\) and \(t\).

**Hint.** Consider \(\frac{d}{dt} \left[ \frac{1}{R(t)} \right]^2 \).

b. When does the circle extinct? (i.e., Find the time \(T\) such that \(R(T) = 0\).)