These problems are designed to help with mistakes that were commonly made on Midterm 2.

On problem 2 (the optimization problem) one common mistake was that the volume was differentiated incorrectly.

**Problem 1.** The volume $V$ of a cube with side length $\ell$ is increasing with time. Write an expression for the rate of change of the volume of this cube in terms of $\ell$. What rule from calculus ensures that your answer has an $\ell'$ in it?

On problem 3 (the graphing problem) one common mistake was in finding asymptotes.

**Problem 2.** Write down a limit condition that implies that the graph of a function $f(x)$ has a vertical asymptote at $x = a$. Your answer should not involve the number 0.

Another similar issue was with finding horizontal asymptotes. Two different limits must be checked when doing this.

**Problem 3.** Find the horizontal asymptotes of the function

$$\frac{2x^3}{|x^3| + 3}$$

In problem 3 on the midterm, another common mistake was that the second derivative of

$$\frac{x^2}{x^2 + 1}$$

was not fully simplified, making it hard to solve for when that second derivative is zero. As practice, here is a problem which could give rise to a similar issue:

**Problem 4.** Compute the second derivative of the function

$$\frac{x^3}{x^3 + 1}$$

using the quotient rule and fully simplify your answer. There should be an $(x^3 + 1)^3$ on the bottom of your answer, and not an $(x^3 + 1)^4$. 


The second derivative of the function \( \frac{x^2}{x^2 + 1} \) is, in unsimplified form, \(-6x^4 - 4x^2 + 2\) \(\frac{1}{(x^2 + 1)^4}\).

Even in this unsimplified form, it is possible to use the quadratic formula to solve for when this second derivative is zero. The technique is to solve for \(x^2\) using the quadratic formula and then take the square root of that.

**Problem 5.** Solve the equation

\[ 0 = -6x^4 - 4x^2 + 2 \]

by writing

\[-6x^4 - 4x^2 + 2 = -6(x^2)^2 - 4(x^2) + 2\]

and using the quadratic formula to solve for \(x^2\), and then taking square roots to solve for \(x\). Note that there are only two real solutions.

On the fourth problem, many had trouble applying l’Hospital’s rule to a product. There are two ways you can do this. If the limit

\[ \lim_{x \to a} f(x)g(x) \]

is an indeterminate form of type \(0 \cdot \infty\), then you could try applying l’Hospital’s rule to

\[ \lim_{x \to a} \frac{f(x)}{1/g(x)} \quad \text{or} \quad \lim_{x \to a} \frac{g(x)}{1/f(x)}. \]

This is because

\[ \frac{f(x)}{1/g(x)} = f(x)g(x) = \frac{g(x)}{1/f(x)}. \]

You have to choose which one looks easier.

**Problem 6.** Find

\[ \lim_{x \to \infty} x \sin(1/x) \]

using l’Hospital’s rule. Note that this is an indeterminate form of type \(0 \cdot \infty\).

\[ \ast \ast \ast \]

Homework guidelines, from the syllabus: Please write your name clearly on your homework, and please staple multiple pages. To receive full credit, you must show your work and justify your answers. Please turn in your homework to the box outside the lecture hall. All problems (including “challenge problems”) are weighted equally. Challenge problems will be significantly more difficult than the rest of the homework, and you should not worry if you do not solve them.