
PRACTICE MIDTERM I

by

Mirela Ciperiani

Problem 1

Consider the following systems

$$\begin{array}{l} 2x + 3y + 2z = 5 \\ \text{a) } \quad x - 3y + 2z = -2 \\ \quad 2x + y - z = 3 \end{array}$$

$$\begin{array}{l} \text{b) } \quad 2x + y + 2z = 3 \\ \quad x - 3y + 2z = -1 \end{array}$$

Do the following for each of the above systems.

- i) Write the coefficient matrix.
- ii) Write the augmented matrix.
- iii) Find the row echelon form of the augmented matrix.
- iv) Find the reduced row echelon form of the augmented matrix.
- v) Identify the leading (basic) and free variables.
- vi) Compute the rank and nullity of the coefficient matrix.
- vii) Find the solution set (i.e. the general solution).

Problem 2

Are the following systems consistent or not?

$$\begin{array}{l} \text{a) } \begin{array}{rcl} x & - & y + 2z = 5 \\ x & + & 2y - z = 1 \\ & - & 3y + z = 2 \end{array} \end{array}$$

$$\text{b) } \begin{array}{rcl} x & - & y = 1 \\ 3x & + & 2y = 8 \end{array}$$

Problem 3

Consider the following matrices:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ -1 & -2 & -3 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 7 \\ 1 & 27 \\ 5 & -2 \end{pmatrix}$$

- i) Compute AB and BA if they are defined.
- ii) Find the transpose of B , the transpose of A , and their product $B^T A^T$.
- iii) Are the columns of A linearly independent?
- iv) Is $Ax = b$ consistent for every $b \in \mathbb{R}^3$?
- v) Do the columns of B^T span \mathbb{R}^2 ?

Problem 4

Compute the inverses of A, B, C, and AC if they exist.

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 2 \\ 0 & 2 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 6 \\ 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 4 \end{pmatrix}$$

Problem 5

Prove that if A is in row echelon form then its nonzero rows are linearly independent.

February 13, 2007

MIRELA CIPERIANI