Linear Algebra - Midterm IIB

Your Name:

Do not open this until you are told to do so.

Please turn off your cell phone.

Calculators, books, class notes and formula sheets are **NOT** allowed in this exam.

All your answers must be accompanied by calculations or/and explanations.
5 problems

Good luck!

Problem 1	/22=3+5+4+6+4
Problem 2	/9
Problem 3	/5
Problem 4	/20=4+3+3+10
Problem 5	/11=4+5+2
Total points	/67

Problem 1

Consider the following linear transformation.

$$\mathrm{T}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 \\ x_2 - x_3 \\ x_1 + x_3 \end{pmatrix}$$

a) Find the standard matrix A of T.
b) Determine if T is invertible.
c) Find a basis for the range of T.
d) Determine the matrix representation of T with respect to the following basis of $\mathbb{R}^3$

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \right\}.$$

e) What is the determinant of $[\mathrm{T}]_{\mathcal{B}}$?

Problem 2

Let $\mathrm{T} : \mathbb{R}^n \to \mathbb{R}^m$ be a one-to-one linear transformation and $\{v_1, \ldots, v_k\}$ a linearly independent subset of $\mathbb{R}^n$. Prove that $\{\mathrm{T}(v_1), \ldots, \mathrm{T}(v_k)\}$ is linearly independent.

Problem 3

Let A be a diagonalizable $n \times n$. Prove that A^3 is also diagonalizable.

Problem 4

Consider the matrix:

$$A = \begin{pmatrix} -3 & 0 & -2 \\ -6 & c & -2 \\ 1 & 0 & 0 \end{pmatrix}$$

Determine all the values of the scalar c for which A is diagonalizable. (Justify all your claims carefully.)

Problem 5

Let $\mathcal{B} = \{b_1, b_2, b_3\}$ where:

$$b_1 = \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}, b_2 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}, b_1 = \begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}.$$

a) Show that $\mathcal{B}$ is a basis for $\mathbb{R}^3$.
b) Determine the matrix $\mathrm{A} = \big([e_1]_{\mathcal{B}}[e_2]_{\mathcal{B}}[e_3]_{\mathcal{B}}\big)$.
c) What is the relationship between A and $\mathrm{B} = \big([b_1]_{\mathcal{B}}[b_2]_{\mathcal{B}}[b_3]_{\mathcal{B}}\big)$