Linear Algebra - Midterm II

Your Name:

Do not open this until you are told to do so.

Please turn off your cell phone.

Calculators, books, class notes and formula sheets are **NOT** allowed in this exam.

All your answers must be accompanied by calculations or/and explanations.
5 problems

Good luck!

Problem 1	/22=3+5+4+6+4
Problem 2	/7
Problem 3	/8
Problem 4	/20=4+3+3+10
Problem 5	/10=4+6
Total points	/67

Problem 1

Consider the following linear transformation.

$$\mathrm{T}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_1 + x_2 - x_3 \\ x_2 + x_3 \\ x_1 + 2x_2 \end{pmatrix}$$

a) Find the standard matrix A of T.
b) Determine if T is invertible.
c) Find a basis for the range of T.
d) Determine the matrix representation of T with respect to the following basis of $\mathbb{R}^3$

$$\mathcal{B} = \left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \right\}.$$

e) What is the determinant of $[\mathrm{T}]_{\mathcal{B}}$?

Problem 2

Let v and w be two eigenvectors of the linear operator $\mathrm{T} : \mathbb{R}^n \to \mathbb{R}^n$ with distinct corresponding eigenvalues. Prove that the set $\{v, w\}$ is linearly independent.

Problem 3

Let U and V be two subspaces of $\mathbb{R}^m$. Prove that $\mathrm{U} \cap \mathrm{V}$ is a subspace of $\mathbb{R}^m$.

Problem 4

Consider the matrix:

$$A = \begin{pmatrix} 0 & 0 & -2 \\ -4 & c & -4 \\ 4 & 0 & 6 \end{pmatrix}$$

Determine all the values of the scalar c for which A is diagonalizable. (Justify all your claims carefully.)

Problem 5

Let $\{v, w\}$ be a basis of the subspace W of $\mathbb{R}^n$, and define

$$z = w - \frac{v \cdot w}{v \cdot v} v$$

Prove that $\{z, v\}$ is an orthogonal basis of W.