Linear Algebra - Midterm IB

Your Name:

Do not open this until you are told to do so.

Please turn off your cell phone.

Show all your work on this booklet.
The blue books are for scratch work only.

Calculators, books, class notes and formula sheets are **NOT** allowed in this exam.

All your answers must be accompanied by calcutations or/and explanations.
5 problems

Good luck!

Problem 1	/28=3+2+3+3+4+2+2+5+4
Problem 2	/6
Problem 3	/8=3+2+3
Problem 4	/4
Problem 5	/8
Total points	/54

Problem 1

Consider the following matrices:

$$A = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & -2 & 4 \\ 1 & 2 & -4 & 11 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & -2 \\ 1 & 2 & -4 \end{pmatrix}$$

a) Compute the reduced row echelon form of A.

b) Verify that BC = A where C denotes the reduced row echelon form of A.

c) Determine $C^T B^T$.

d) Consider the system of linear equations with A as its augmented matrix. Determine if the system is consistent. If so, find its solutions.

e) Determine the rank and nullity of A (explain your answer).

f) Determine whether the equation $Ax = v$ has a solution for every $v \in \mathbb{R}^3$.

g) Determine whether the equation $Ax = v$ has at most one solution for every $v \in \mathbb{R}^3$.

h) Consider the system $Ax = 0$. Determine the free and leading variables. Find its general solution and write it in vector form.

i) Compute B^{-1} and A^{-1} if possible. Verify that $BB^{-1} = I_3$.

Problem 2

Find a polynomial function $f(x) = ax^2 + bx + c$ whose graph passes through the points $(1, 3)$, $(-1, 1)$, and $(-2, 3)$.

Problem 3

Consider the set $\mathcal{S} = \{\mathrm{v}_1, \mathrm{v}_2, \mathrm{v}_3\}$ where

$$\mathrm{v}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \qquad \mathrm{v}_2 = \begin{pmatrix} 1 \\ -2 \\ -4 \end{pmatrix} \qquad \mathrm{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$$

a) Determine whether the set $\mathcal{S}$ is linearly independent.

b) Determine whether the vector

$$\mathrm{v} = \begin{pmatrix} 2 \\ -3 \\ -5 \end{pmatrix}$$

is in the span of $\mathcal{S}$.

c) Determine whether the set $\mathcal{S}$ spans $\mathbb{R}^3$.

Problem 4

Let A and B be two $n \times n$ matrices. Prove that if B and AB^{-1} are invertible, then A is invertible.

Problem 5

Let A and B be two symmetric matrices such that AB is symmetric. Prove that AB = BA.