

## Linear Algebra - Midterm I

**Your Name:**

Do not open this until you are told to do so.

Please turn off your cell phone.

Show all your work on this booklet.

The blue books are for scratch work only.

Calculators, books, class notes and formula sheets are  
**NOT** allowed in this exam.

**All your answers must be accompanied by  
calculations or/and explanations.**

**5 problems**

Good luck!

Problem 1	/28=3+2+3+3+4+2+2+5+4
Problem 2	/6
Problem 3	/8=3+2+3
Problem 4	/4
Problem 5	/8
Total points	/54

## Problem 1

Consider the following matrices:

$$A = \begin{pmatrix} 1 & 2 & 1 & 6 \\ 0 & 1 & 3 & 6 \\ -1 & 1 & 9 & 13 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 3 \\ -1 & 1 & 9 \end{pmatrix}$$

- a) Compute the reduced row echelon form of A.
- b) Verify that  $BC = A$  where C denotes the reduced row echelon form of A.
- c) Determine  $C^T B^T$ .
- d) Consider the system of linear equations with A as its augmented matrix. Determine if the system is consistent. If so, find its solutions.
- e) Determine the rank and nullity of A (explain your answer).
- f) Determine whether the equation  $Ax = v$  has a solution for every  $v \in \mathbb{R}^3$ .
- g) Determine whether the equation  $Ax = v$  has at most one solution for every  $v \in \mathbb{R}^3$ .
- h) Consider the system  $Ax = 0$ . Determine the free and leading variables. Find its general solution and write it in vector form.
- i) Compute  $B^{-1}$  and  $A^{-1}$  if possible. Verify that  $BB^{-1} = I_3$ .

**Problem 2**

Find a polynomial function  $f(x) = ax^2 + bx + c$  whose graph passes through the points  $(1, 2)$ ,  $(-1, 4)$ , and  $(2, 4)$ .

**Problem 3**

Consider the set  $\mathcal{S} = \{v_1, v_2, v_3\}$  where

$$v_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad v_2 = \begin{pmatrix} 1 \\ 2 \\ 6 \end{pmatrix} \quad v_3 = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$$

- a) Determine whether the set  $\mathcal{S}$  is linearly independent.
- b) Determine whether the vector

$$v = \begin{pmatrix} 2 \\ 1 \\ 6 \end{pmatrix}$$

is in the span of  $\mathcal{S}$ .

- c) Determine whether the set  $\mathcal{S}$  spans  $\mathbb{R}^3$ .

**Problem 4**

Let  $A$  and  $B$  be two  $n \times n$  symmetric matrices. Prove that  $A + B$  is symmetric.

### Problem 5

Let  $A$  be an  $m \times n$  matrix in row echelon form. Prove that the nonzero rows of this matrix are linearly independent.

