• The use of class notes, book, formulae sheet, calculator is not permitted.

• In order to get full credit, you must:
  a) get the correct answer, and
  b) show all your work and/or explain the reasoning that lead to that answer.

• Each solution must have a clearly labeled problem number and start at the top of a new page.

• Please make sure the solutions you hand in are legible and lucid. You may only use techniques we have developed in class.

• You have one hour and fifteen minutes to complete the exam.

• Do not forget to write your name and UNI in the space provided below and on the top of each page.

Enjoy the exam, and good luck!
Exercise 1. [10 points] Fix a triangle $T$ as in the picture defined by two edges of size $x$ and $y$, and an angle $a$ between them. Assume that the edge $x$ is increasing at the rate of $3 \text{ cm/sec}$, $y$ is decreasing at the rate of $2 \text{ cm/sec}$ and $a$ is increasing at the rate of $0.5 \text{ rad/sec}$. Find the rate of change of the area of the triangle $T$ when $x = 40 \text{ cm}$, $y = 50 \text{ cm}$ and $a = \pi/6 \text{ rad}$.

Exercise 2. [15 points] Evaluate the limit or show that it does not exist:

a) $\lim_{(x,y)\to(0,0)} \frac{2x \sin y}{x^2 + 2y^2}$, 

b) $\lim_{(x,y)\to(0,0)} (x^2 + y^2) \ln(x^2 + y^2)$, 

c) $\lim_{(x,y)\to(1,0)} \frac{e^y(xy - y)}{\sqrt{x^2 + y^2 - 2x + 1}}$.

Exercise 3. [20 points] Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by the formula

$$f(x,y) = \begin{cases} 0 & \text{if } y \leq 0 \text{ or } y \geq x^2, \\ 1 & \text{if } 0 < y < x^2. \end{cases}$$

a) Show that $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along any line through $(0,0)$.

b) Despite part (a), show that $f$ is discontinuous at $(0,0)$.

c) Show that $f$ is discontinuous along two entire curves and write these curves explicitly.

Exercise 4. [10 points] The ellipsoid $4x^2 + 3y^2 + 2z^2 = 16$ intersects the plane $y = 2$ in an ellipse. Find the parametric equations of the tangent line to this ellipse at the point $(1, 2, 0)$.

Exercise 5. [15 points] Consider the surface $yz + x \ln(y) - z^2 = 0$.

a) Find $\partial z/\partial x$ and $\partial z/\partial y$.

b) What is the domain of the function $z$? When is the function $z$ differentiable?

Exercise 6. [30 points] True/False. Justify your answer with a proof if true, or a counterexample if false.

a) For two parametric curves $\mathbf{r}_1(t)$ and $\mathbf{r}_2(t)$ we have $\frac{d(\mathbf{r}_1(t) \times \mathbf{r}_2(t))}{dt} = \mathbf{r}_1(\prime)(t) \times \mathbf{r}_2(\prime)(t)$.

b) The expression $1 + y/3$ gives a linear approximation to the function $\sqrt{y + \cos^2x}$ near the point $(0,0)$.

c) The domain of the function $f(x,y) = (\sin^{-1}(x^2 + y^2)) \ln(1 + x^2), \frac{e^x}{x + y - 1})$ is obtain by removing the line $x = 1 - y$ from the disc of radius 1 centered at $(0,0)$.

d) The derive $F_{\alpha,\alpha}$ of the function $F(\alpha,\beta) = \int_{\beta}^{\alpha^2} \sqrt{1 + t^3} \, dt$ equals $\frac{2 + 5\alpha^3}{\sqrt{1 + \alpha^3}}$.

e) The cross derivatives of the function $f(x,y) = x^3 e^y - \sin(y) \ln(1 + x^2)$ match.

f) The angle of intersection of the curves $\mathbf{r}_1(t) = \langle t, t^2, t^3 \rangle$ and $\mathbf{r}_2(t) = \langle \sin 2t, \sin 4t, 2t \rangle$ is $\pi/2$. 

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