2. The two numbers are \( x + 100 \) and \( x \). Minimize \( f(x) = (x + 100)x = x^2 + 100x \). \( f'(x) = 2x + 100 = 0 \) \( \Rightarrow \) \( x = -50 \).

Since \( f''(x) = 2 > 0 \), there is an absolute minimum at \( x = -50 \). The two numbers are 50 and -50.

16. \( V = lwh \) \( \Rightarrow \) 10 = \( (2w)(w)h = 2w^2h \), so \( h = \frac{5}{w^2} \).

The cost is \( 10(2w^2) + 6[2wh + 2(hw)] = 20w^2 + 36wh \), so
\[
C(w) = 20w^2 + 36w\left(\frac{5}{w^2}\right) = 20w^2 + 180/w.
\]
\[
C'(w) = 40w - 180/w^2 = 40\left(w^2 - \frac{9}{2}\right)/w^2 \Rightarrow w = \sqrt{\frac{9}{2}} \text{ is the critical number. There is an absolute minimum for } C \text{ when } w = \sqrt{\frac{9}{2}} \text{ since } C'(w) < 0 \text{ for } 0 < w < \sqrt{\frac{9}{2}} \text{ and } C'(w) > 0 \text{ for } w > \sqrt{\frac{9}{2}}.
\]
\[
C\left(\sqrt{\frac{9}{2}}\right) = 20\left(\sqrt{\frac{9}{2}}\right)^2 + \frac{180}{\sqrt{9/2}} \approx 163.54.
\]

20. The distance \( d \) from the point \((3, 0)\) to a point \((x, \sqrt{x})\) on the curve is given by \( d = \sqrt{(x - 3)^2 + (\sqrt{x} - 0)^2} \) and the square of the distance is \( S = d^2 = (x - 3)^2 + x \). \( S' = 2(x - 3) + 1 = 2x - 5 \) and \( S' = 0 \) \( \iff \) \( x = \frac{5}{2} \). Now \( S'' = 2 > 0 \), so we know that \( S \) has a minimum at \( x = \frac{5}{2} \). Thus, the \( y \)-value is \( \sqrt{\frac{5}{2}} \) and the point is \( \left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right) \).

48. In isosceles triangle \( AOB \), \( \angle O = 180^\circ - \theta - \theta \), so \( \angle BOC = 2\theta \). The distance rowed is \( 4\cos \theta \) while the distance walked is the length of arc \( BC = 2(2\theta) = 4\theta \). The time taken is given by \( T(\theta) = \frac{4\cos \theta}{2} + \frac{4\theta}{4} = 2\cos \theta + \theta, 0 \leq \theta \leq \frac{\pi}{2} \).

\[
T'(\theta) = -2\sin \theta + 1 = 0 \iff \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}.
\]

Check the value of \( T' \) at \( \theta = \frac{\pi}{6} \) and at the endpoints of the domain of \( T \); that is, \( \theta = 0 \) and \( \theta = \frac{\pi}{2} \).

\( T(0) = 2, T\left(\frac{\pi}{6}\right) = \sqrt{3} + \frac{\pi}{6} \approx 2.26, \text{ and } T\left(\frac{\pi}{2}\right) = \frac{\pi}{2} \approx 1.57. \) Therefore, the minimum value of \( T \) is \( \frac{\pi}{2} \) when \( \theta = \frac{\pi}{6} \); that is, the woman should walk all the way. Note that \( T''(\theta) = -2\cos \theta < 0 \) for \( 0 \leq \theta < \frac{\pi}{2} \), so \( \theta = \frac{\pi}{6} \) gives a maximum time.

63. Here \( s^2 = h^2 + b^2/4 \), so \( h^2 = s^2 - b^2/4 \). The area is \( A = \frac{1}{2} b \sqrt{s^2 - b^2/4} \).

Let the perimeter be \( p \), so \( 2s + b = p \) or \( s = (p - b)/2 \) \( \Rightarrow \)
\[
A(b) = \frac{1}{2} b \sqrt{(p - b)^2/4 - b^2/4} = b \sqrt{p^2/2 - 2pb/4}. \text{ Now}
\]
\[
A'(b) = \frac{\sqrt{p^2 - 2pb} - \frac{bp}{4}}{\sqrt{p^2 - 2pb}} = \frac{-3pb + p^2}{4 \sqrt{p^2 - 2pb}}.
\]

Therefore, \( A'(b) = 0 \) \( \Rightarrow \) \( -3pb + p^2 = 0 \) \( \Rightarrow \) \( b = p/3 \). Since \( A'(b) > 0 \) for \( b < p/3 \), and \( A'(b) < 0 \) for \( b > p/3 \), there is an absolute maximum when \( b = p/3 \). But then \( 2s + p/3 = p \), so \( s = p/3 \) \( \Rightarrow \) \( s = b \) \( \Rightarrow \) the triangle is equilateral.
7. \[ f(x) = x^5 - x - 1 \implies f'(x) = 5x^4 - 1, \text{ so } x_{n+1} = x_n - \frac{x_n^5 - x_n - 1}{5x_n^4 - 1}. \] Now \( x_1 = 1 \implies x_2 = 1 - \frac{1 - 1 - 1}{5 - 1} = 1 - \left(\frac{1}{4}\right) = 1.25 \implies x_3 = 1.25 - \frac{(1.25)^5 - 1.25 - 1}{5(1.25)^4 - 1} \approx 1.1785.

11. To approximate \( x = \sqrt[5]{20} \) (so that \( x^5 = 20 \)), we can take \( f(x) = x^5 - 20 \). So \( f'(x) = 5x^4 \), and thus,

\[ x_{n+1} = x_n - \frac{x_n^5 - 20}{5x_n^4}. \] Since \( \sqrt[5]{32} = 2 \) and 32 is reasonably close to 20, we’ll use \( x_1 = 2 \). We need to find approximations until they agree to eight decimal places. \( x_1 = 2 \implies x_2 = 1.85, x_3 \approx 1.82148614, x_4 \approx 1.82056514, \]
\[ x_5 \approx 1.82056420 \approx x_6. \] So \( \sqrt[5]{20} \approx 1.82056420 \), to eight decimal places.

Here is a quick and easy method for finding the iterations for Newton’s method on a programmable calculator.
(The screens shown are from the TI-84 Plus, but the method is similar on other calculators.) Assign \( f(x) = x^5 - 20 \) to \( Y_1 \), and \( f'(x) = 5x^4 \) to \( Y_2 \). Now store \( x_1 = 2 \) in \( X \) and then enter \( X - Y_1/Y_2 \rightarrow X \) to get \( x_2 = 1.85 \). By successively pressing the ENTER key, you get the approximations \( x_2, x_4, \ldots \).

In Derive, load the utility file \texttt{SOLVE}. Enter \texttt{NEWTON} \((x^5 - 20, x, 2)\) and then \texttt{APPROXIMATE} to get \[ [2, 1.85, 1.82148614, 1.82056514, 1.82056420]. \] You can request a specific iteration by adding a fourth argument. For example, \texttt{NEWTON} \((x^5 - 20, x, 2, 2)\) gives \[ [2, 1.85, 1.82148614]. \]

In Maple, make the assignments \( f := x \rightarrow x^5 - 20; \ g := x \rightarrow x - f(x)/D(f)(x); \) and \( x := 2; \). Repeatedly execute the command \( x := g(x) \); to generate successive approximations.

In Mathematica, make the assignments \( f[x_] := x^5 - 20; \ g[x_] := x - f[x]/f'(x); \) and \( x = 2 \). Repeatedly execute the command \( x = g[x] \) to generate successive approximations.

36. \( f(x) = x \cos x \implies f'(x) = \cos x - x \sin x \). \( f'(x) \) exists for all \( x \), so to find the maximum of \( f \), we can examine the zeros of \( f' \). From the graph of \( f' \), we see that a good choice for \( x_1 \) is \( x_1 = 0.9 \). Use \( g(x) = \cos x - x \sin x \) and \( g'(x) = -2 \sin x - x \cos x \) to obtain \( x_2 \approx 0.860781, x_2 \approx 0.860334 \approx x_4 \).

Now we have \( f(0) = 0, f(\pi) = -\pi, \) and \( f(0.860334) \approx 0.561096 \), so \( 0.561096 \) is the absolute maximum value of \( f \) correct to six decimal places.
39. We need to minimize the distance from $(0, 0)$ to an arbitrary point $(x, y)$ on the curve $y = (x - 1)^2$. \[ d = \sqrt{x^2 + y^2} \implies d(x) = \sqrt{x^2 + [(x - 1)^2]^2} = \sqrt{x^2 + (x - 1)^4} \]. When $d' = 0$, $d$ will be minimized and equivalently, $s = d^2$ will be minimized, so we will use Newton’s method with $f = s'$ and $f' = s''$.

\[ f(x) = 2x + 4(x - 1)^9 \implies f'(x) = 2 + 12(x - 1)^2, \text{ so } x_{n+1} = x_n - \frac{2x_n + 4(x_n - 1)^2}{2 + 12(x_n - 1)^2} \]. Try $x_1 = 0.5$ \implies $x_2 = 0.4, x_3 \approx 0.410127, x_4 \approx 0.410245 \approx x_6$. Now $d(0.410245) \approx 0.537841$ is the minimum distance and the point on the parabola is $(0.410245, 0.347810)$, correct to six decimal places.