Eilenberg Lectures

Speaker: Alex Lubotzky

Title: Subgroup Growth

Tuesday, 4-5:15 PM Room 507 Mathematics

Let $\Gamma$ be a finitely generated group, $a_n(\Gamma)$ = the number of subgroups of $\Gamma$ of index $n$ and $\xi_\Gamma(s) = \sum_{n-1}^{\infty} a_n(\Gamma) n^{-s}$. The lectures will deal with the relationship between the algebraic properties of $\Gamma$, the asymptotic behavior of $\{a_n(\Gamma)\}_{n=1}^{\infty}$ and the analytical properties of $\xi_\Gamma(s)$. We compare "subgroup growth" to "word growth" and the zeta functions of groups to Dedekind zeta functions of ring of algebraic integers.

Lecture 1. September 12

Overview; free groups

Basic definitions will be presented as well as an overview of the subject. The case of free groups will be presented.

Lecture 2. September 19

On groups of polynomial subgroup growth

The main theorem characterizing the groups with $a_n(\Gamma) \leq n^c$ will be presented. The proof involves diverse methods: the classification of finite simple groups, Hilbert 5th problem for $p$-adic Lie groups, the theory of algebraic and arithmetic groups and the prime number theorem.

Lecture 3. September 26

Subgroup growth and the congruence subgroup problem

The asymptotic of the number of congruence subgroups in arithmetic groups will be determined ("non-commutative analytic number theory"). It will be shown that the congruence subgroup property can be expressed in these terms and therefore has meaning also for non-arithmetic lattices in semi-simple Lie groups.

Lecture 4. October 10

Counting hyperbolic manifolds

The congruence subgroup problem for fundamental groups of 3-dimensional hyperbolic groups will be answered. An application will be presented to the counting of $n$-dimensional hyperbolic manifolds as a function of their volume.

Lecture 5. October 17

Counting normal subgroups; counting maximal subgroups; intermediate growth and gap results

The analogous questions for normal/maximal subgroups will be studied. Some new ingredients come up - such as probability! We consider also the possible growth types

Lecture 6. October 24

Zeta functions of groups

The zeta functions of nilpotent groups will be studied. The rationality of the local $p$-part will be shown using methods from model theory and $p$-adic integration. The global properties will be studied using counting points on varieties over finite fields.