

**MATH W4051 PROBLEM SET 5**  
**DUE OCTOBER 8, 2008.**

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- (1) Munkres 30.3.
- (2) Munkres 30.4.
- (3) Munkres 30.13.
- (4) Let  $M$  be a topological manifold. Prove:
  - (a)  $M$  is first-countable.
  - (b) If  $M$  is Hausdorff then  $M$  is regular.
- (5) Munkres 31.1
- (6) Munkres 31.2
- (7) Munkres 31.5
- (8) Munkres 32.3
- (9) Prove: given a topological space  $X$ , there is a topological space  $X_H$ , the *Hausdorffification* of  $X$ , together with a map  $\pi: X \rightarrow X_H$  such that:
  - $X_H$  is Hausdorff and
  - Given any Hausdorff topological space  $Y$  and continuous map  $f: X \rightarrow Y$  there is a unique map  $f_H: X_H \rightarrow Y$  so that  $f = g \circ \pi$ , i.e.,

$$\begin{array}{ccc} X & \xrightarrow{f} & Y \\ \pi \downarrow & \nearrow f_H \exists! & \\ X_H & & \end{array}$$

(Hint: construct  $X_H$  as a quotient space of  $X$ .)

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