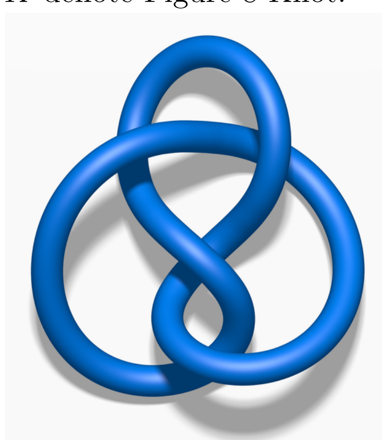
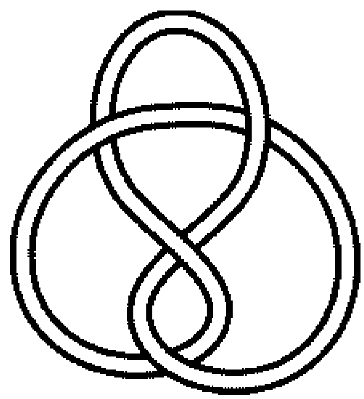
MATH G4307 PROBLEM SET 3
DUE SEPTEMBER 27, 2011.

INSTRUCTOR: ROBERT LIPSHITZ

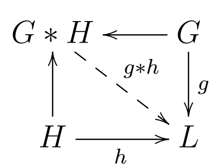
Exercises to turn in:

(E1) Hatcher 1.2.4.
(E2) Hatcher 1.2.9.
(E3) Hatcher 1.2.16. Do this two ways. First, use Hatcher's version of Van Kampen's theorem where he allows covers by infinitely many open sets. Second, use the version of the Seifert-van Kampen theorem for two sets. (Hint for the second: $[0,1]$ and $[0,1]\times[0,1]$ are compact.)
(E4) Hatcher 1.2.22. And:
(c) Let K denote Figure 8 Knot:





Compute $\pi_1(\mathbb{R}^3 \setminus K)$.
(d) For K the Figure 8 Knot, show that there is a homomorphism from $\pi_1(\mathbb{R}^3\setminus K)$ to D_5, the group of symmetries of a regular pentagon. Use this to conclude that K is genuinely knotted.
(E5) Recall the universal mapping property for free products of groups: given groups G and H there is a group $G*H$ and maps $G\to G*H$, $H\to G*H$ so that for any other group L and maps $g\colon G\to L$, $h\colon H\to L$ there is a unique map $(g*h)\colon G*H\to L$ so that



commutes.

What is the analogue for free products with amalgamation? Prove your claim.

Problems to think about but not turn in:

(P1) Recall that we defined $H^1(X) = [X, S^1]$. What's the analogue of our weak van Kampen theorem (i.e., $\pi_1(X \vee Y) \cong \pi_1(X) * \pi_1(Y)$) for H^1? Of the full Van Kampen theorem?

(P2) How does Exercise (E5) relate to wedge sums? What's the analogue of free products with amalgamation in other categories (e.g., sets, topological spaces, based topological spaces)? Can you give a very abstract statement of Van Kampen's theorem?

(P3) Read through the remaining problems in this section, and do any that seem difficult, surprising or interesting. (There are lots of very nice exercises in this section.)

E-mail address: `lipshitz@math.columbia.edu`