Holomorphic curves and stable homotopy theory

Soren Galatius
(Stanford)
December 7, 2007

Let $M(X)$ denote the moduli space of (nodal, stable, genus $g$) curves holomorphic curves in $X$. Invariants (e.g. Gromov-Witten) can be extracted from two ingredients: (Parts of) the cohomology ring $H^*(M(X))$, and the fundamental class $H^*(M(X)) \to \mathbb{Q}$. I will describe a space $F(X)$ and a natural map $u : M(X) \to F(X)$. From the point of view of stable homotopy theory, the definition of the space $F(X)$ is a rather natural construction, and in particular the rational cohomology ring of $F(X)$ can be easily and explicitly described. All relevant cohomology classes in $M(X)$ arise as pull back from classes in $F(X)$. This framework collects all the "homotopy theoretic" information into one object, and all the "analytic" information is encoded in the fundamental class $H^*(F(X)) \to \mathbb{Q}$. In the case where $X$ is a point (my talk will focus on this case), the fundamental class "is" the power series determined by Kontsevich’s theorem (Witten’s conjecture). This is joint work with Ya. Eliashberg.

New York Area Joint Symplectic Geometry Seminar
5:15 p.m.
Math 312
Columbia University