FURTHER SMALL CORRECTIONS AND EXPLANATIONS FOR "A CYLINDRICAL REFORMULATION OF HEEGAARD FLOER HOMOLOGY"

ROBERT LIPSHITZ

This document contains further mild corrections and explanations for [Lip06], beyond those corrected in [Lip14].

Page 26 of [Lip14]. In the proof of Proposition 4.2′, the instances to $\pi_{\mathbb{D}} \circ u$ should be $\pi_\Sigma \circ u$. (Thanks to Morgan Weiler for pointing out this typo.)

Page 1001 of [Lip06] In Proposition 8.6, the proof that A_ζ induces an action of the exterior algebra is incorrect: the moduli spaces $\widehat{\mathcal{M}}^A_{K,2}$ have an unaccounted for end where $p_1 \to p_2$. To correct the proof, we first show that for any $\zeta, \eta \in H_1(Y)$, the map $A_\zeta \circ A_\eta + A_\eta \circ A_\zeta = 0$ on $\widehat{HF}$, HF^+, HF^-, and HF^∞. To see this, choose disjoint knots $K_\zeta, K_\eta \subset \Sigma \times [0,1]$ representing ζ and η, and consider the index 2 moduli space of holomorphic curves with one point mapped to ζ and a second point mapped to η. The ends of this moduli space show that $A_\zeta \circ A_\eta + A_\eta \circ A_\zeta$ is chain homotopic to the zero map. Next, to see that $A_\zeta^2 = 0$ on Floer homology it suffices to consider the case that ζ is represented by a chain K in Σ which is dual to some α_i, i.e., K intersects α_i in one point and is disjoint from α_j for $j \neq i$. Let K' be a small isotopic translate of K, and consider the moduli space of holomorphic curves

$$\{u\colon (S,p,q) \to \Sigma \times [0,1] \times \mathbb{R} \mid \pi_\Sigma(u(p)) \in K,\ \pi_\Sigma(u(q)) \in K',\ \pi_{\mathbb{R}}(u(p)) - \pi_{\mathbb{R}}(u(q)) > 0\}$$

(and with u satisfying the conditions (M0)–(M6) from the paper). This moduli space has no end with $\pi_{\mathbb{R}}(u(p)) - \pi_{\mathbb{R}}(u(q)) \to 0$ because K and K' are disjoint and intersect the α-circles in a single point. Then, it is easy to see that the ends of the moduli space imply that A_ζ^2 is chain homotopic to 0.

(Thanks to Ian Zemke for pointing out this mistake.)

Page 1005 of [Lip06]. In the proof of Lemma 9.3, the fact that the ends of $\overline{\widehat{\mathcal{M}}_1(\vec{x}^1, \vec{y}^2, k)}$ correspond to height 2 holomorphic buildings in which the $\mathbb{R}$-invariant level has ind $= 1$ and the non-$\mathbb{R}$-invariant level has ind $= 0$ is not sufficiently justified, because Proposition 4.2 was only proved with respect to $\mathbb{R}$-invariant almost complex structures. The easiest solution is to define Φ to only count embedded, rigid holomorphic curves in homology classes with ind $= 0$. (This is, in some sense, three conditions: the combinatorial index $\mathrm{ind}(A) = e(A) + n_{\vec{x}}(A) + n_{\vec{y}}(A) = 0$, the curve must be embedded, and the curve must lie in a 0-dimensional moduli space. Presumably the condition that $\mathrm{ind}(A) = 0$ implies the other two, but that has not been shown for non-$\mathbb{R}$-invariant almost complex structures.) Similarly, define $\overline{\widehat{\mathcal{M}}_1(\vec{x}^1, \vec{y}^2, k)}$ to consist of $\mathrm{ind}(A) = 1$, 1-dimensional moduli spaces of embedded curves with $n_{\mathfrak{z}} = k$. Since $\mathrm{ind}(A)$ agrees with the dimension of the moduli space of curves

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for $\mathbb{R}$-invariant levels and is additive under gluing, if a sequence of curves in $\overline{\widehat{\mathcal{M}}_1(\vec{x}^1, \vec{y}^2, k)}$ converges to a 2-story holomorphic building then the $\mathbb{R}$-invariant level must have $\mathrm{ind}(A) = 1$, so the non-$\mathbb{R}$-invariant level must have $\mathrm{ind}(A) = 0$. Note also that gluing preserves (non-)embeddedness. It follows that the ends of $\bigcup_k \overline{\widehat{\mathcal{M}}_1(\vec{x}^1, \vec{y}^2, k)}$ correspond to the terms in $\partial \circ \Phi + \Phi \circ \partial$, as desired. (Thanks to Cagatay Kutluhan for pointing out this gap.)

References

[Lip06] Robert Lipshitz, *A cylindrical reformulation of Heegaard Floer homology*, Geom. Topol. **10** (2006), 955–1097, arXiv:math.SG/0502404.

[Lip14] ———, *Correction to the article: A cylindrical reformulation of Heegaard Floer homology [mr2240908]*, Geom. Topol. **18** (2014), no. 1, 17–30.