REMARK ON THE PAPER ""ACTIONS OF FINITE CYCLIC GROUPS ON QUASICOMPLEX MANIFOLDS"

UDC 513.836

I. M. KRİČEVER

In the author's paper [1] two theorems were formulated (Theorems 1.11 and 2.2), asserting that certain conditions on a collection of $Z_m$-bundles were necessary and sufficient for this collection to be the collection of normal bundles to the fixed submanifolds of a unitary $Z_m$-manifold. Unfortunately, these conditions are in fact only sufficient. However, the methods of [1] allow one, at the cost of purely technical complications, to obtain necessary and sufficient conditions.

We consider the category whose objects are collections $(X, \{\xi_i\})$, where $X$ is a $G$-manifold and $\{\xi_i\}$ is a finite collection of $G$-bundles over $X$. The bordism groups $U^G_{n, \mu}$ in this category ($n = \dim_p X$, $\mu = (\mu_1, \ldots)$) is a "multi-index" set $\mathbb{R}^\infty$ replace for us the bordism groups of $G$-manifolds, which, incidentally, are the special case of $U^G_{n, \mu}$ for $\mu_i = 0$.

For a collection of $Z_p^k$-bundles $\{\xi_i\}$ over $X$ (prime), let $\xi_{i_1}$ be the restriction of $\xi_i$ to a fixed submanifold $F_{i_1}$ of $X$, and $\nu_{i_1}$ the normal bundle of $F_{i_1}$ in $X$. As usual, the collection of $Z_p^k$-bundles $\nu_{i_1}$, $\{\xi_{i_1}\}$ over the trivial $Z_p^k$-manifolds $F_{i_1}$ defines a homomorphism

$$p^k : U^Z_{n, \mu} \rightarrow R_{n, \mu} = \sum U_{nl} \left( \prod_{j=1}^{k-1} BU(n_j) \times \prod_i BU(n_i, i) \right),$$

where the sum is taken over those collections of nonnegative integers $(l, \{n_j\}, \{n_{i, j}\})$ for which $2(\sum_j n_j + l) = n$ and $\sum_j n_{i, j} = \mu_i$.

We denote by $\Psi : R_{*, p^k} \rightarrow R_{*, p^k}$ the homomorphism induced by the change of indices $(i, j) \rightarrow (pi + s, j') (0 \leq s \leq p - 1, j \equiv s (\mod p^k), j' = (j - s)/p)$, and also $j \rightarrow (s, j')$.

For $\omega = (i_1, \ldots, i_n)$, let $\nu_{i_1}^\omega (u_1, \ldots, u_n)$ be the series obtained by symmetrization of the series $\sum_{i=1}^n u_i s [CP(u_s)]^{-1}$, where $CP(u) = \sum_0^\infty [CP^m] u^m$. For each collection $\omega_i$ of length $\mu_i$ there is defined a homomorphism $\nu_{\omega_i}$, whose value on the additive generator $[M] \times \Pi_{i,j} [CP^m] (\nu_{i,j}^\omega (u_{i,j}, \ldots, u_n))$ of the group $U_{n, \mu} \prod_{j=0}^{k-1} BU(n_j, i_1)$ is equal to $[u]^N [M] \times \Pi_{i,j} [CP^m] (\nu_{i,j}^\omega (u_{i,j}, \ldots, u_n))$ by replacing $u_k^k$ by $[CP^{m_{s-k}}]$. $\nu_{\omega_i}$.

We define homomorphisms $D\alpha_j$ as follows: if $(j, p) = 1$, then

AMS (MOS) subject classifications (1970). Primary 55C35, 57D85; Secondary 57D90.
\[ D \alpha_j \left( [M] \times \prod_{s=1}^{n_j} (CP_j^m) \right) = \left[ u \right]_{p_j}^m \cdot [M] \cdot \prod_{s=1}^{n_j} \left( \frac{u}{\left[ u \right]_{p_j}^s} \right)^{m_{s+1}} B_{m_s}(\left[ u \right]_s), \]

where \( \dim_{\mathbb{C}} [M] = m \); but if \( p \) divides \( j \), then

\[ D \alpha_j \left( [M] \times \prod_{s=1}^{n_j} (CP_j^m) \right) = \left[ u \right]_{p_j}^{m-1} \cdot [M] \cdot \prod_{s=1}^{n_j} \left( \frac{u}{\left[ u \right]_{p_j}^s} \right)^{m_{s+1}} B_{m_s}(\left[ u \right]_s). \]

We denote by \( [D \alpha]_{\omega} \odot = (\omega_1, \ldots) \), the tensor product of the homomorphisms \( D \alpha_j \) and \( V_{\omega_i} \).

\[ [D \alpha]_{\omega} : R_{Z^{n_i, u}} \rightarrow U^* \left( \left[ u \right] / \theta_p \left( \left[ u \right]_{p_j}^{k-1} \right) \right) = 0. \]

We introduce an additional graduation in \( R^{Z_{p_j}^k} \), by making each collection \( \{ n_{i,j} \} \) correspond to the number \( d = \sum_{(i,j) = 1} n_{i,j} \). Now let \( \rho_d \) be the "homogeneous component" of \( \rho \in R^{Z_{p_j}^k} \).

**Theorem.** A bordism class \( \rho \in R^{Z_{p_j}^k} \) belongs to \( \text{Im} \beta^k \) if and only if \( \Psi(\rho) \in \text{Im} \beta^{k-1} \) and for any \( \omega \), the quantity

\[ \sum_{d=0}^{n} \left( \frac{u}{\left[ u \right]_{p_j}^d} \right)^{n-d} [D \alpha]_{\omega}(\rho_d) \]

is divisible by \( u^n \) in the ring \( U^* \left( \left[ u \right] / \theta_p \left( \left[ u \right]_{p_j}^{k-1} \right) \right) = 0. \)

To describe \( \text{Im} \beta^{Z_{p_j}^m} \) in the case when \( m \) is divisible by at least two primes, it is necessary to insert an analogous correction into the hypothesis of Theorem 2.2 of [1].

Received 20/MAR/74

**BIBLIOGRAPHY**


Translated by N. STIEK