Introduction to Knot Theory
Exercises #5

Due Dec. 10

1. Compute the Jones polynomials for the prime knots of 6-crossings or less.
2. Show that 8_{19} has no alternating diagram. i.e. that it is inequivalent to any knot of fewer crossings and that it has no alternating diagram of 8 or more crossings.
3. Show that 9_{33} is chiral.
4. Suppose that K is alternating and amphichiral. Show that the minimal number of crossings for a diagram of K, $c(K)$, is even.
5. Any two alternating diagrams for a knot can be connected by a series of *flypes*. (See pg. 246 of the handout or http://en.wikipedia.org/wiki/Flype for a picture.) The knot 8_{17} is negative amphichiral (equivalent to it's reversed mirror). Find the corresponding series of flypes. **Change: Do the same for the diagrams for 5_2 in Figure 1.**
6. Let D be an alternating diagram for K with C crossings. Checkerboard color the regions in the plane cut out by D so that the A-regions are colored black. Let W and B be the number of white and black regions respectively (including the infinite face). Define the "black graph" to be a vertex for every black region and an edge between regions exactly when there is a crossing connecting them.
   (a) Show that reduced diagrams are exactly those for which the black graph does not have edges which begin and end at the same vertex.
   (b) Show that the all A resolution gives an unlink with W components and the B resolution an unlink with B components.
   (c) Show that the all A resoultion, S, contributes the term of maximal degree to $\langle D\rangle$.[1] Hint: If S' has only one B resoution, $|S'| = |S| - 1$. What happens to the leading term of $\langle D|S''\rangle$ for S'' with more than one B resoution?
   (d) Show that the term of minimal degree in $\langle D\rangle$ is contributed by the all B resoution. Hint: Consider the mirror.
   (e) Using the above, conclude that:

$$\max\deg\langle D\rangle = C + 2(W-1) \text{ and } \min\deg\langle D\rangle = -C - 2(B-1)$$

[1] Recall that $\langle D|S\rangle = A^{\#A-\#B \text{ resultions}}(-A^2 - A^{-2})^{|S|-1}$

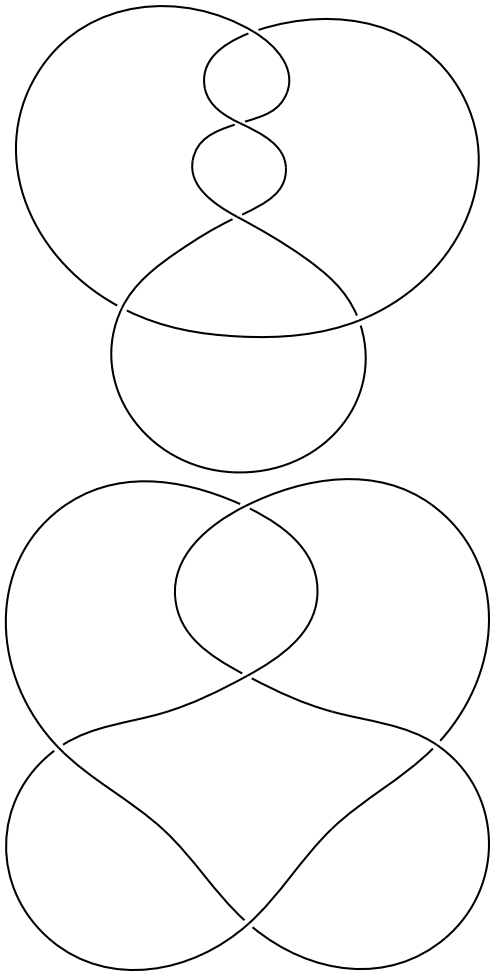

Figure 1: Two diagrams for 5_2.